Chapter 4 Bonus Material

— Introduction —

Are you someone who wishes there were more examples, discussions, and commentaries in the intentionally brief descriptions of the lessons? If so, you have come to the right place! This file contains bonus material for some of the activities from chapter 4.

For puzzles, many examples of solved puzzles are given, along with additional commentary on how to create them. The Early Family Math program is based on the idea that early mathematics is something a family should do together, and making puzzles for your child to do with you is an important part of that process. Once you get the hang of each puzzle, you should find that most if not all the puzzles are fairly easy for you to create.

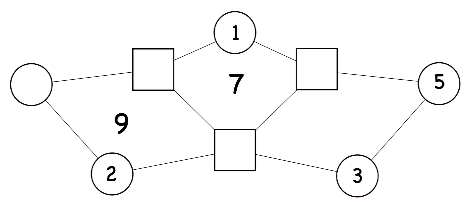
Many of these puzzles have different levels of difficulty, and there are many suggestions and examples in the coming pages for how to create those levels. Always start with the easiest puzzles. It is far better to have your child experience success, understanding, and fun with puzzles that are a bit too easy, than to be frustrated, discouraged, and over-challenged by puzzles that are too hard. Once your child builds confidence and enthusiasm for a math activity, that is the time to slowly incorporate greater challenges. Also, not all puzzles will be fun for everyone, so don't push puzzles and activities that just don't seem to connect.

This is what you will find in the following pages:

Chapter 4 - Enclosed Sums

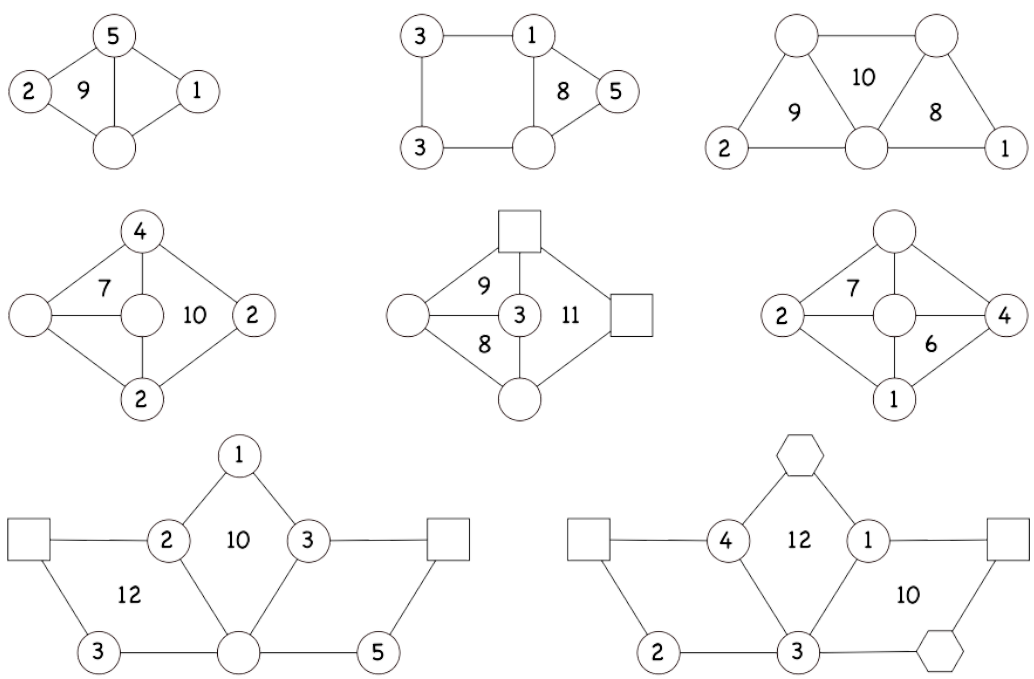
These puzzles have shapes connected by lines. Each enclosed region has a number that is the sum of the shapes that border it. Similar to Shape Sums puzzles, circles may have any value, and the value for a non-circular shape must be the same as any other shape of the same type. For example, all squares must have the same value and all hexagons would have the same value. You can optionally add the rule that different non-circular shapes must have different values - for example, that squares and hexagons must have different values.

The puzzle for your child is to figure out the numbers in the shapes and regions that are not supplied.



Create these puzzles by making a diagram of circles and maybe some other shapes. Next, fill in all the figures with numbers and fill in the bounded regions with the sum of the figures that surround them. Finally, remove some of the numbers.

As with Shape Sums puzzles in Chapter 3, start with simple puzzles with just one or two numbers missing and slowly progress to puzzles with more numbers missing, more enclosed regions next to each other, and more use of values in non-circular regions.



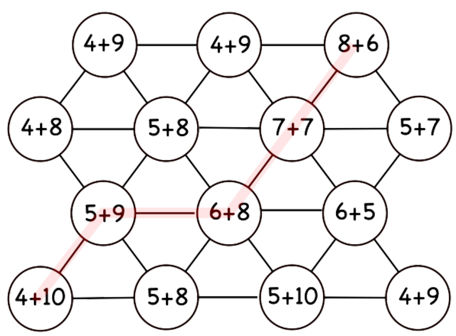
Chapter 4 - Island Hopping - Compensation

Using compensation for addition is a way to make addition problems much easier. The idea is to take an amount away from one of the numbers being added and give it to the other other number - the result stays the same, but one of the numbers becomes easier to work with.

For example, when you add 7 + 8, if you take 2 away from 7 and give it to the 8, the problem becomes 5 + 10. Alternatively, if you take 3 away from the 8 and give it to the 7, the problem becomes 10 + 5. Anytime you can make one of the numbers a multiple of 10, you will have a much simpler problem.

These puzzles provide practice in creating new problems using compensation. The challenge is to find a path that connects all the islands with the same answer. It is only legal to connect two islands if their problem's numbers differ by 1. Only some of the islands will be on the path.

Make these puzzles by starting with about ten islands with some connections. Identify a path from one edge of the islands to the other. Along that path, put problems that differ from each other by one - perhaps start with a problem that involves adding 10, and then make variations on it. In the islands near to the path, put problems with small changes that have different answers.

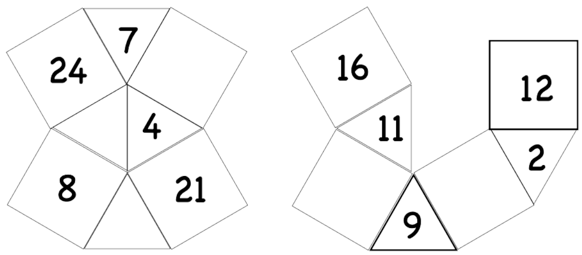


There really is little to be done to vary the hardness of these puzzles. Introducing false paths will probably lead to confusion rather than challenge, and so it is generally a bad idea.

Chapter 4 - DiffTriangles and SumTriangles

— DiffTriangles —

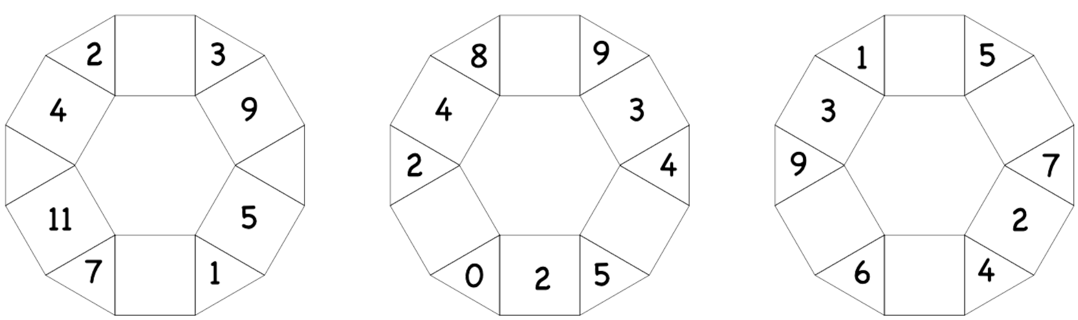
DiffTriangles puzzles have triangles and squares that share sides. A triangle always has exactly two squares on its sides, and the remaining side has either a triangle or is empty. A triangle's number is the difference of the two adjoining squares. The challenge is to supply the missing numbers.



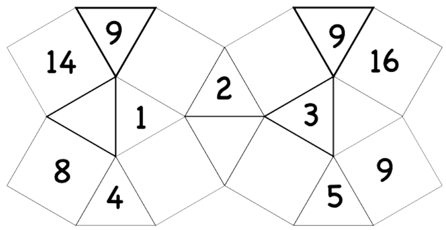
Constructing Puzzles: Making puzzles without loops is easy. Draw an alternating sequence of squares and triangles, put in numbers starting at one end, and then work your way to the far end. When you are done, remove some of the numbers. Making puzzles with loops or more complicated interactions is trickier; however, the effort pays off with some challenging puzzles!

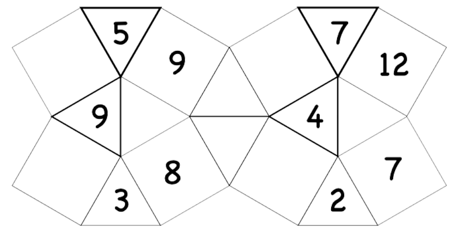
When your child gets very comfortable with these, they may want to take a turn creating some new puzzles of their own. They should have fun and learn a lot by figuring out how the numbers fit together.

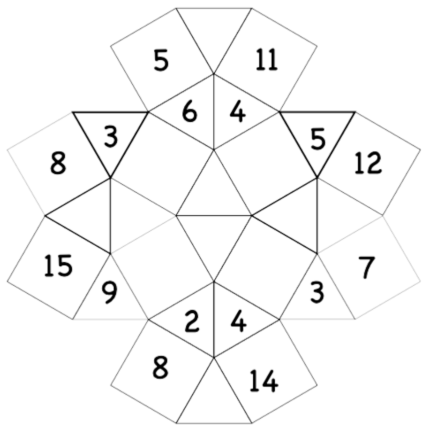
Strategies for Solving: The places to do first are any triangles between two filled in squares. Another easy case is a square next to a filled triangle that has a smaller filled square next to it - in this case, because we are not working with negative numbers, there is only one choice for filling in the empty square. The most common case is a square that has two possible values looking in one direction, and two other possibilities looking in the other direction - there is usually only one number that overlaps in those possibilities.

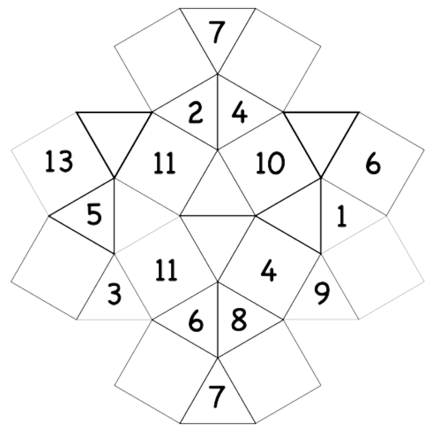


Here are some examples with lots of interconnections.



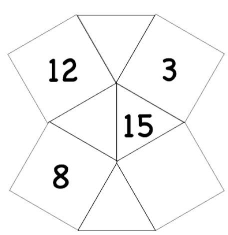


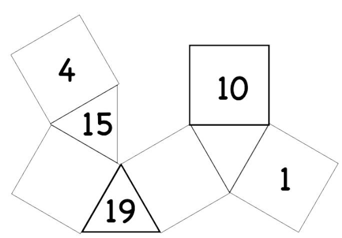


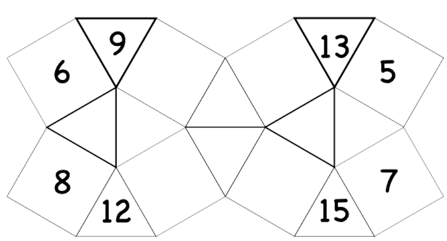


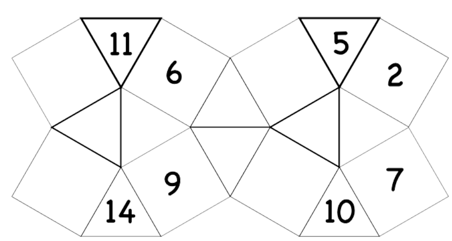
— SumTriangles —

SumTriangles puzzles are just like DiffTriangles only they use addition in place of subtraction. The value of a triangle is the sum of its two or three square neighbors. Make these puzzles using methods similar to DiffTriangles. SumTriangles puzzles are typically simpler to solve than DiffTriangles.



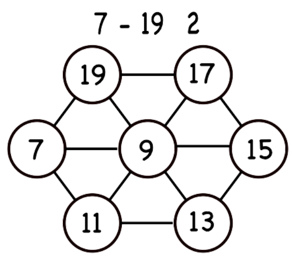


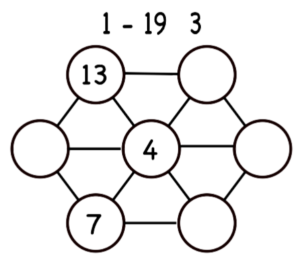




Chapter 4 - Island Hopping - Skip Counting

These puzzles have islands (circles) connected by bridges (lines). In this version of Island Hopping, the connections are made by skip counting. Some of the islands have numbers written on them and some will start off blank. Above the puzzle is the starting number, ending number, and the skip amount. The challenge is to fill





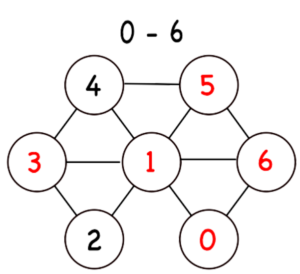
in the missing numbers and find the path. You can also place the numbers and blanks on pieces of paper on the floor to make a stepping puzzle.

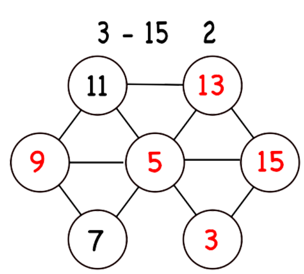
As with the Skip Counting activity, create puzzles to practice going forward or backward starting at a variety of numbers, not just numbers that are a multiple of the skip amount.

Creating these puzzles is the same as creating the Island Hopping - Counting puzzles from early in Chapter 2. Make the islands first, fill in the skip counting numbers, connect those islands in the correct order, and then add some additional connections to help make a puzzle out of it. In the version you give your child, remove some numbers leaving enough of the numbers so that it can still be figured out.

You can revisit the puzzle construction strategies described in the Bonus Material for Chapter 2 for Island Hopping - Counting. Also, if you still have any of those puzzles, it is very easy to convert one of those puzzles to one of these. Take the following puzzle from Chapter 2. It involves counting from 0 to 6. The red numbers are the ones that would normally be left out when the puzzle is given to your child. To convert it to a puzzle that starts at 3 and skip counts by 2, simply multiply all the numbers by 2 and then add 3 to them, as in the table below. After that, replace the original numbers by the new ones (leaving out the red ones, of course).

	0	1	2	3	4	5	6
Mult. by 2	0	2	4	6	8	10	12
Add 3	3	5	7	9	11	13	15





Chapter 4 - Fix It

Start with a 4 by 4 grid of numbers with a target sum. The challenge is to find entries to remove so that the sum of the remaining numbers in every row and column is the target. An alternative version uses individual target sums for each row and column.

Make these puzzles by putting in pairs or triples of numbers that sum to the target sum. Then fill in the remaining spaces with decoy numbers. You can make these trickier by having alternative pairs or triples of numbers that partially work. If your child is enjoying these, but finding them too easy, you can always make larger ones that are 4 by 5, 5 by 5, or even larger.

Red stars have been added here to show which entries would be removed to make the puzzles work.

8

6	3	5	2
2	1	4	5
3	4	1	3
6	4	2	5

9

7	4	5	2
2	1	4	6
3	4	4	1
6	4	5	3

10

3	3	6	4
7	1	2	6
4	6	1	4
6	4	8	2

11

8	3	5	4
1	1	4	7
3	8	1	3
7	5	7	4

Here are two puzzles using individual target sums for the rows and columns.

6	3	7	8	16
2	1	4	5	9
3	4	7	3	10
5	6	3	5	11
11	9	18	8	

0	6	5	2	8
7	8	5	4	12
2	7	1	4	9
3	1	9	8	17
9	13	14	12	

Chapter 4 - Island Hopping By Ones and Tens

A rectangular grid of numbers is given with some of the numbers filled in. The challenge is to fill in the remaining numbers so that any two numbers that share a side only differ in a single place, and the difference of the digits in that place is 1 (including going between 0 and 9). No number may be used more than once in the entire grid. Referencing a 100-Chart may be helpful for beginning solvers.

Make this puzzle by taking an empty grid and filling it with numbers, with no number repeated. Next, remove some of the numbers, making sure that it is not too hard for your child. In these examples, the red numbers are the missing ones.

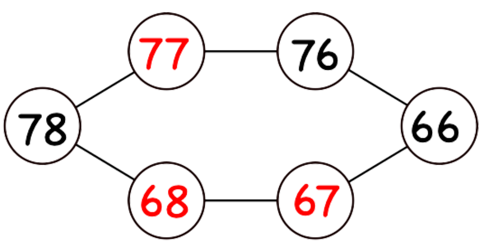
57	67	66	56

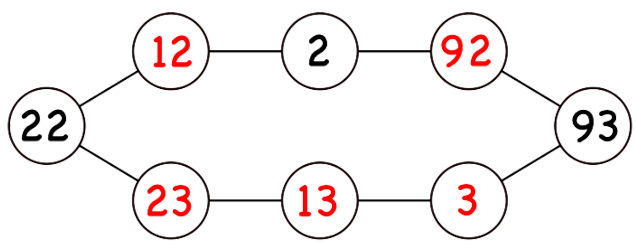
5	4	94	95

33	23	13
32	22	12

Using only one-digit and two-digit numbers, there is not a lot of trickiness that can be introduced. However, they are great practice for thinking about place value. One wrinkle that may surprise your child are transitions such as 95 to 5 to 15 or 11 to 10 to 0 to 9 - they may not realize there is a 0 in the tens place for single-digit numbers and they may be surprised by 0 and 9 being connected.

Grids are a natural way to present these problems. However, the puzzles can also be represented in the same way as other Island Hopping puzzles using circles, and this representation allows for some additional freedom in creating puzzles.





Chapter 4 - Solitaire Shape Puzzles

— Magic Triangles —

Make a triangle of six circles with three circles on a side. In the circles, use each of the numbers from 1 to 6 once so that each side of the triangle has the same sum. This involves two challenges - finding out which sums will work and then figuring out how to get those sums. It is better to let your child play with this to figure out which sums are possible, but if frustration wins out, the possible sums are 9, 10, 11, and 12.

If your child enjoys figuring this out, this can be done for larger triangles as well. For a triangle with nine circles with four circles on a side, the possible sums are 17, 19, 20, 21, and 23.

As with so many of the puzzles for this age group, the main reason to have your child play with this is to encourage having fun exploring how numbers interact with each other and to practice number facts. They do not yet have the math or reasoning skills to be systematic with their exploration. However, these puzzles can be explored more deeply, and here are some ideas to dig into if you or an older child are interested.

Let SUM represent the sum of one side of the triangle. If you add up the three sides of the triangle, the total will be 3 x SUM. However, the total of the three sides will also be the sum of all the numbers plus one extra copy for each corner of the triangle. Let C-SUM be the sum of the values in the three corners. We end up with the relationship that 3 x SUM = (Total of all the numbers) + C-SUM.

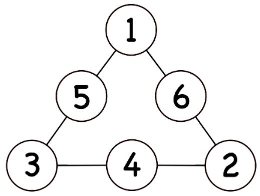
6 circle puzzle. Apply this to the triangle with six circles. The sum of all the numbers is the sum of the numbers from one to six, which is 21. So the equation becomes 3 x SUM = 21 + C-SUM The smallest C-SUM can be is 1 + 2 + 3 = 6, and the largest it can be is 4 + 5 + 6 = 15. So, 3 x SUM is between 21 + 6 = 27 and 21 + 15 = 36. This forces SUM to be 9, 10, 11, 12. Note also that C-SUM = 3 x SUM - 21, which is handy for finding the corners.

Another thing to notice is the symmetry of the possible values. What is causing this symmetry is that for every solution, there is another solution created by subtracting all the numbers from 7 (or from 10 for the nine circle puzzle). A little calculating will show that this symmetry takes a puzzle with sum SUM and creates a new one with sum (21 - SUM) (or 40 - SUM for the nine circle puzzle).

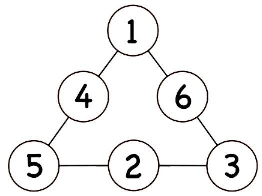
The last thing to notice before we dig in with actual numbers is that for any solution for the three corners, we can assume that they are in increasing order going around clockwise, with the smallest number at the top. If they are not in that configuration to begin with, you can rotate or flip the diagram until they are.

All these observations save a tremendous amount of work. We only need to look at SUM equal to 9 and 10, and we only need to have the corners in increasing order. If SUM is 9, then C-SUM = 3 x 9 - 21 = 6, so the trio is 1, 2, and 3. If SUM is 10, then a + b + c = 3 x 10 - 21 = 9. This leaves two possibilities - either corner values of 1, 2, and 6, or 1, 3, and 5. A quick trial rules out 1, 2, and 6 as a possibility.

After much work, we have the solutions for SUM being 9 and 10 for the six circle puzzle. Remember that you can get the solutions for SUM being 11 and 12 by subtracting all the entries from 7.



Sum = 9



Sum = 10

<u>9 circle puzzle</u>. Use the same approach for the 9 circle puzzle. The sum of the numbers from 1 to 9 is 45. Hence, 3 x SUM = 45 + C-SUM. The smallest C-SUM can be is 1 + 2 + 3 = 6, and the largest it can be is 7 + 8 + 9 = 24. So 3 x SUM is between 45 + 6 = 51 and 45 + 24 = 69, which forces SUM to be between 17 and 23. Taking a solution and subtracting all the entries from 10 gives the following SUM pairings: 17 - 23, 18 - 22, 19 - 21, and 20 - 20. So, solutions are only needed for 17, 18, 19, and 20. The corresponding values for C-SUM are 6, 9, 12, and 15.

SUM = 17 and C-SUM = 6. For this, the corners must be 1, 2, 3, and it works.

SUM = 18 and C-SUM = 9. For this, the corners must be either 1, 2, 6 or 1, 3, 5. Neither works.

SUM = 19 and C-SUM = 12. There are quite a few possibilities for the corners, but the only combinations that work are 1, 4, 7 and 2, 3, 7.

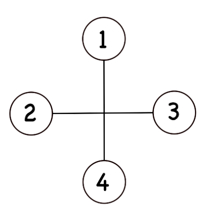
SUM = 20 and C-SUM = 15. There are too many combinations for the corners, and many of them work. Two that work are 1, 5, 9 and 2, 5, 8.

— Magic Designs —

Similar to Magic Triangles, these have circles connected in a geometric pattern and an associated group of numbers. Put the numbers in the circles so every straight line of connected circles has the same sum.

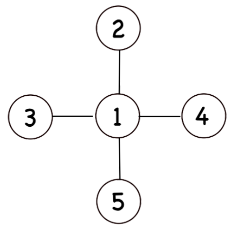
The analysis of these puzzles is similar to what was done for Magic Triangles. Let SUM be the common sum that all the rows share. Let c be the value of the middle circle, for puzzles that have one. The general strategy will be to add up all the rows and investigate the relationship that is revealed. Note also that, just as for Magic Triangles, a new solution can be created by subtracting all the entries from one more than the largest number.

1. The numbers from 1 to 4 are in a plus sign shape with no circles in common. The numbers 1 to 4 add up to 10, and this is split evenly between the two directions. So SUM = 5 and the answer is easy.

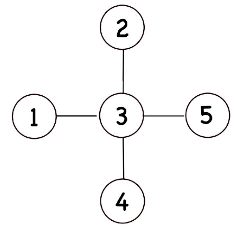


SUM = 5

2. The numbers from 1 to 5 are in a plus sign with one circle in common in the middle. The numbers 1 to 5 add up to 15. Adding up the two directions gives 2 x SUM = 15 + c. Because 15 + c must be even, c can be 1, 3, and 5. Get the solution for c = 5 (SUM = 10) from the c = 1 solution by subtracting all the numbers from 6.

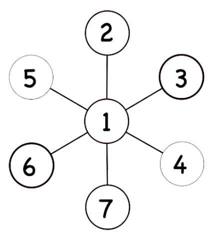


c = 1 SUM = 8

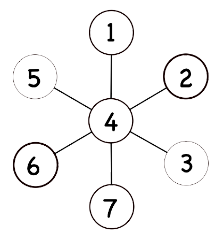


c = 3 SUM = 9

3. The numbers from 1 to 7 are in lines of 3 circles with one common circle in the middle. Adding up the three directions gives 3 x SUM = 28 + 2 x c. Because 3 evenly divides 28 + 2 x c, this forces c to be 1, 4, or 7. The solutions for c = 1 and 4 are given.

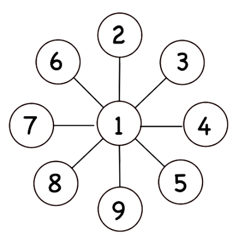


c = 1 SUM = 10

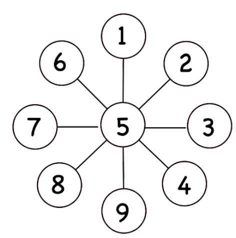


c = 4 SUM = 12

4. The numbers from 1 to 9 are in lines of 3 circles with one common circle in the middle. Adding up the four directions gives 4 x SUM = 45 + 3 x c. Because 4 evenly divides 45 + 3 x c, this forces c = 1, 5, or 9.



c = 1 SUM = 12

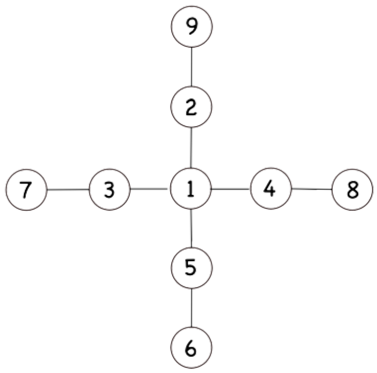


c = 5 SUM = 15

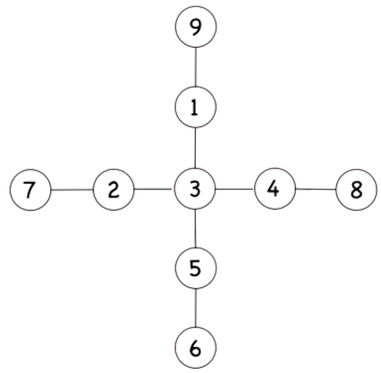
5. The numbers from 1 to 5 are placed in an L shape with one circle in common in the corner. This is really the same as problem #2, so the solutions are essentially the same.

6. The numbers from 1 to 8 are in a plus sign with no circles in common. The two directions evenly split 36, the sum of all the numbers, so SUM = 18. There are many ways to solve this by splitting the set of numbers into two groups that add up to 18. One solution is 1, 2, 7, 8 and 3, 4, 5, 6, and another is 1, 3, 6, 8 and 2, 4, 5, 7.

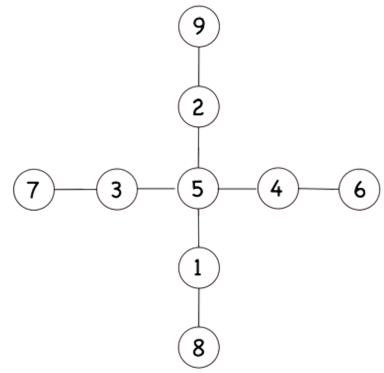
7. The numbers from 1 to 9 are in a plus sign with one circle in common in the middle. Adding up the two directions gives 2 X SUM = 45 + c, so c = 1, 3, 5, 7, and 9. Solutions for c = 1, 3, and 5 are given.



c = 1 SUM = 23

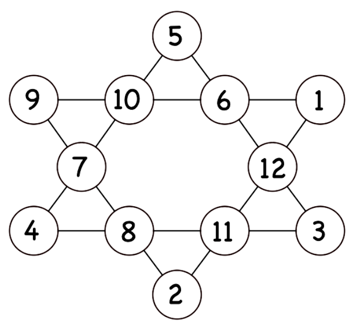


c = 3 SUM = 24

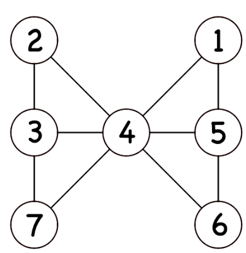


c = 5 SUM = 25

8. The numbers from 1 to 12 are in a star shape. This has 6 directions of lines of 4 circles. This one is much harder than the others. If you add up all the directions, every number will be involved twice. The numbers from 1 to 12 add up to 78. Thus we have 6 x SUM = 2 x 78, which means SUM = 26 (as given in the hint). A solution is given below. As always, another solution can be obtained by subtracting all the entries from 13.



9. The numbers from 1 to 7 are in an H shape - 3 vertically on the left, 1 in the center, 3 vertically on the right. There are 5 possible lines of 3 connected circles. If the 5 directions are added up, all the circles will be used twice, with the exception of the center which is used three times. Adding up the five directions gives 5 x SUM = 2 x 28 + c. Because 5 evenly divides 56 + c, this forces c = 4, and in that case SUM = 12 (as given in the hint). Note that neither 2 nor 3 can be on the same side as the 1, and this leads to the following solution.



Chapter 4 - Sum Square

Start with a 3 by 3 grid that has target sums given for each row and column. Some of the numbers from 1 to 9 are already placed in the grid. For the numbers that are not yet placed, the challenge is to place them to make the row and column sums be the target values.

To make one of these puzzles, start by placing pieces of paper with the numbers from 1 to 9 on a 3 x 3 grid. For each row and column, write the sum to the right or below. Then, remove some of the numbers from the grid. Lastly, hand the pieces of paper you removed to your child and ask “where were these?” Because these are so easy to create, they are great puzzles for your child to create for you to solve.

One variation that keeps the sums a little smaller is to use the numbers from 0 to 8 instead. A harder variation is to do the same thing with the numbers 1 to 12 in a 3 by 4 grid, or even 1 to 16 in a 4 by 4 grid.

6	3	5	14
2	8	4	14
7	1	9	17
15	12	18	

6	3	5	14
2	8	4	14
7	1	9	17
15	12	18	

6	3	5	14
2	8	4	14
7	1	9	17
15	12	18	

Making the original filled in puzzle is easy enough. As mentioned above, just put in all the numbers and write down the sums. The challenge for the puzzle maker is to remove just the right amount of information so that the puzzle is challenging but not too hard.

<u>Strategies for Solving and Creating</u>: Start by filling in squares that are the single missing numbers in a row or column. The leftmost of these three puzzles is pretty easy to solve because, after the 5 and the 7 are filled in, then the 3 and 2 are easy to solve, and then lastly the 8 will be easy- solving each singleton creates new singletons that are easy to calculate.

Easy to calculate puzzles are good practice for your child, so don’t worry about making all the puzzles tricky.

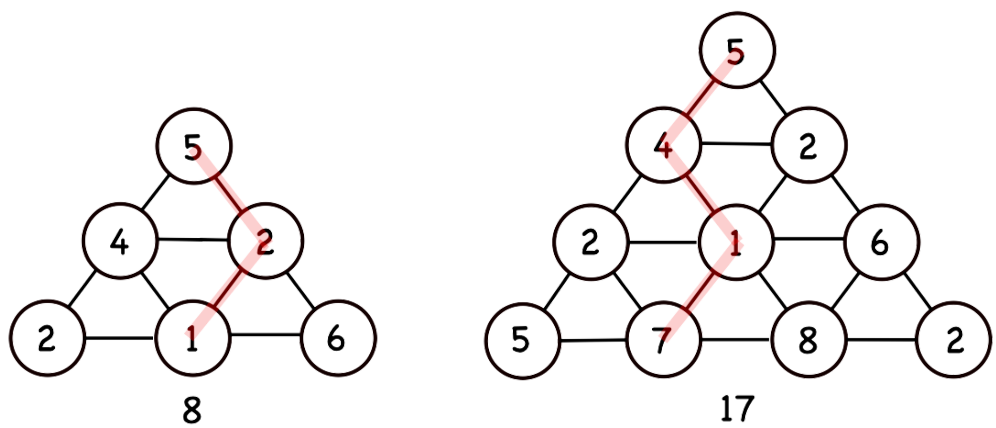
The middle puzzle is a bit harder. There are no singletons. A good strategy for these is to look for rows or columns that have particularly large or small missing sums - these will have relatively few choices to choose from. The bottom row and the rightmost column are good places to start for this puzzle. The missing numbers in the bottom row add up to 16, so they must be 7 and 9. The 9 cannot go in the column with the 6 (the sum would be too big for that column), so that places the 7 and 9. The rest follows as in the previous puzzle.

In the rightmost puzzle, two of the side numbers are left out. Once your child realizes that the side numbers add up to 45, which is the sum of the numbers from 1 to 9, it is easy to fill in a single missing side number.

Chapter 4 - Addition Pyramid

A pyramid of 10 numbers placed in 4 rows is given with a target number. The challenge is to find a path through the pyramid using one number from each row so that the sum of the numbers is the target number. The numbers on the path must touch each other.

Make one of these puzzles by filling in the numbers that you want to form the path, and record the sum of those numbers. Then fill in the remaining decoy numbers in the pyramid. The number of possible paths through the pyramid doubles with the addition of each row, so making larger pyramids is a way to challenge a child who finds the 10-number puzzle easy. For a child who finds a 10-number puzzle hard, start with 6-number puzzles until they become easy and quick to solve.



For larger puzzles, it can be a challenge for the puzzle maker to ensure that there is only one correct path through the pyramid. Don't concern yourself too much with that. Even though it's nice if there is only one path, your child will enjoy showing you that there is more than one way to solve it.

Chapter 4 - Investigations

— FLOWER PETALS —

INVESTIGATION

In a magical garden there are two kinds of flowers. One has 4 petals and the other has 7 petals. A child was asked to pick some flowers so that the total number of petals was 13. Could it be done? How about 15 petals? For which numbers of petals is it possible? For numbers that are possible, can it be done in more than one way? For example, 32 petals is four 7's and one 4, and it is also eight 4's.

By trying many pairs of numbers, there are lots of examples to play with. For some pairs of numbers there comes a point where all numbers of petals are possible, and for other pairs of numbers there is no such point. For 4 and 7, every number from 18 on is possible. For 3 and 6, there is no point after which all numbers occur.

What is the pattern and what creates that pattern? Those are often questions that come up, and it is where many interesting things happen.

It's easiest to see what happens when some number evenly divides both numbers. Take 3 and 6 for example. Think of these numbers as 1 x 3 and 2 x 3. When you add these numbers together, you will always get some number of 3's. There is no way to add 3's and 6's together to get 10, because 10 is not a multiple of 3.

When 1 is the only number that evenly divides both numbers, there will always come a point where every number can be attained. For 4 and 7, that number is 18. To find that number, subtract 1 from each of the numbers in the pair and multiply those new numbers together. In this case, that gives 3 x 6 = 18. Another interesting facet of this situation is that exactly half of the numbers below 18 will be reachable. Why this works takes some math a bit too sophisticated for a young child; however, it is fun to play with these calculations and your child's experiences with these patterns may suddenly click into place much later on.

— CLIMBING STEPS – HOW MANY WAYS —

INVESTIGATION

Suppose your child likes to take steps two at a time sometimes, but one at a time other times. If your child wants to go up some steps, a natural question is: How many ways can this be done?

For example, for 0 steps there is just one way - you just stand there. For 1 step there is one way - you take a single step. For two steps, you can either take one double step or two single steps.

Your child should carefully count many cases of this and make a table of the results. When there is lots of information, a table often helps organize the information and allow patterns to stand out. The table would look like this (okay, going beyond 6 may require too much patience, but here are the numbers):

0	1	2	3	4	5	6	7	8	9
1	1	2	3	5	8	13	21	34	55

After looking at these numbers, your child may notice that each pair of consecutive numbers adds up to the next number. Why does this happen? These numbers are called Fibonacci Numbers. The rule for creating the official Fibonacci Numbers is that each number is the sum of the previous two. This also happens for the steps. Hmmm ...

Let's look closely at one example - say 5 steps. The 8 possibilities are: 1+1+1+1+1, 1+1+2+1, 1+2+1+1, 2+1+1+1, 2+2+1, 1+1+1+2, 1+2+2, and 2+1+2. The first 5 possibilities use 1 for the last move, and the last 3 possibilities use 2 for the last move. That explains it - you can go up 5 steps by either going up 4 steps and taking 1 more step, or by going up 3 steps and going up 2 more steps. The number of ways of going up 5 steps is exactly equal to the sum of the number of ways of going up 4 steps plus the number of ways of going up 3 steps.

Patterns are often understood by patiently going through examples, organizing the data, looking closely at the data, and digging for explanations of why things happen the way they do. This is a good habit to develop in your child.

— BALANCE SCALE —

INVESTIGATION

A balance scale is a simple device for telling when two things have exactly the same weight. The scale is usually supplied with a set of weights that are used to measure the weight of other objects. There are many interesting investigations you can do if you restrict the weights you are allowed to use.

One Kind of Weight: Suppose you have lots of weights, but they are all the same - say, 5 units. Then the only things you can weigh exactly are objects that are a multiple of 5 (just like skip counting by 5).

Two Kinds of Weights - One Side: Suppose you have lots of weights that are either 4 units or 7 units and you only use them on one side of the balance. The things you can weigh are the same numbers you found in the flower petal investigation. For 4 and 7, starting at 18 units you can weigh everything exactly. If the weights are 4 units and 6 units, you can only weigh even numbers starting with 4.

Two Kinds of Weights - Both Sides: After doing the investigation with two kinds of weights on one side, your child might be surprised if you ask them to weigh a 3-unit item, or even a 1-unit item, with 4's and 7's. The trick is to put some weights on one side and other weights on the other side. For example, verify an item weighs 3 units by putting it with a 4-unit weight and see that it balances with a 7-unit weight. Similarly, verify an item weighs 1 unit by putting it with a 7-unit weight and see that it balances with two 4-unit weights.

There is an important math theorem called Bezout's Theorem hidden in this investigation. Your child doesn't need to know about that theorem at this point, but isn't it cool that a young child can be playing around with advanced mathematics!

Doubling Weights: What happens if you have one weight each for each of the weights in the doubling progression 1, 2, 4, 8, and 16? How many ways can you weigh something that weighs 13? What is the largest weight you can measure?

After some investigation, you will find that you can weigh everything up to one less than double the highest weight - in this case that is 31. Also, each item you can weigh can only be weighed in one way - for example, 13 = 1 + 4 + 8, and there is no other way to do it. Pretty cool! This situation is related to the binary number system.

Fibonacci Weights: What happens if the weights are in the Fibonacci Numbers? Is there more than one way to weigh some weights? Find a restriction that would cause there to be only one way for each weight.

Suppose you have one each for the weights 1, 1, 2, 3, 5, 8, and 13. With this, 10 = 2 + 3 + 5 = 2 + 8 = 1 + 1 + 3 + 5 = 1 + 1 + 8. What is causing the duplication is that the Fibonacci rule creates more than one way to write the Fibonacci numbers in terms of themselves - for example, 2 = 1 + 1 and 8 = 5 + 3. The way to fix this problem is to insist that you cannot use two Fibonacci numbers that are neighbors of each other in the sequence. When you add that restriction, the only way to get 10 is 2 + 8.