Chapters 1-2 Bonus Material

— Introduction —

Are you someone who wishes there were more examples, discussions, and commentaries in the intentionally brief descriptions of the lessons? If so, you have come to the right place! This file contains bonus material for some of the activities from chapters 1 and 2.

For puzzles, many examples of solved puzzles are given, along with additional commentary on how to create them. The Early Family Math program is based on the idea that early mathematics is something a family should do together, and making puzzles for your child to do with you is an important part of that process. Once you get the hang of each puzzle, you should find that most if not all the puzzles are fairly easy for you to create.

Many of these puzzles have different levels of difficulty, and there are many suggestions and examples in the coming pages for how to create those levels. Always start with the easiest puzzles. It is far better to have your child experience success, understanding, and fun with puzzles that are a bit too easy, than to be frustrated, discouraged, and over-challenged by puzzles that are too hard. Once your child builds confidence and enthusiasm for a math activity, that is the time to slowly incorporate greater challenges. Also, not all puzzles will be fun for everyone, so don't push puzzles and activities that just don't seem to connect.

This is what you will find in the following pages:

- **Chapter 1 – One of These is Not Like the Others**
- **Chapter 2 – Island Hopping – Counting**
- **Chapter 2 – Cutting Symmetric Shapes**
- **Chapter 2 – Nim with 1 and 2**
- **Chapter 2 – Connect the Dots**
- **Chapter 2 – Shape Sudoku**
- **Chapter 2 – Number Sudoku with Jigsaw Patterns**
- **Chapter 2 – Greater Than Sudoku**
- **Chapter 2 – Make Me a Liar**
- **Chapter 2 – 15-Sliding Puzzle**

Chapter 1 - One of These is Not Like the Others

This activity has your child looking at four things and deciding which three share a property that the fourth one does not. The following is a quick list of examples together with explanations. Children often have a fresh way of looking at things and it is well worth listening to them to see if their reasoning is novel yet sound.

There are several ways to present these four items to your child. The easiest for you is to simply say the list. If the items are easy to draw, you can draw them. If the items are hard to draw, you may be able to find photos or drawings in ads or magazines that you can cut out and select from. You may be able to use a single photo with lots of content and point out four things in the photo.

For activities like this, once your child has practiced this for a while and has a solid idea of the activity, it is good to reverse your roles - your child will learn a lot by creating examples for you to solve. As before, their reasoning may be very different from yours, so listen carefully.

— Groups of Four —

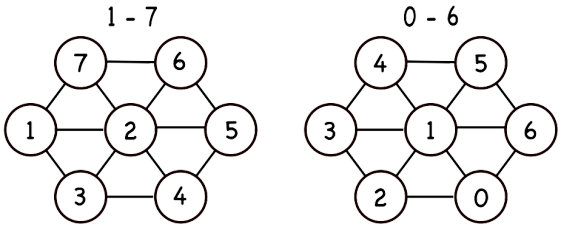
Here are a few examples to help you get started.

- rabbit, dog, butterfly, pillow - The first three are alive and the pillow is not.
- banana, cheese, hammer, carrot - The hammer is the only one you cannot eat.
- cheese, shoes, coat, shirt - The cheese is the only one you cannot wear.
- red triangle, red square with a hole, green square, red square with no hole - Any one of the first three can be the odd one. The red triangle is the only one that is not a square. The red square with a hole is the only one that is not solid. The green square is the only one that is not red.
- dog, cat, lion, goldfish - The lion is the only animal on the list that would make a bad pet. It is also quite a bit bigger than the other animals. Or, three of them have four legs and the fish lives in water.
- rose bush, oak, maple, pine - The rose bush is the only one that is not a tree.
- bench, table, couch, stool - The table is the only one that you don’t sit on. Or only the couch is soft.
- bark, honk, rainbow, click - The rainbow is the only one that is not a sound.
- socks, pants, toothbrush, hat - The toothbrush is the only one that you wouldn’t wear.
- chair, umbrella, sofa, stool - The umbrella is the only one you wouldn’t want to sit on.
- ants, pig, spiders, grasshoppers - The pig is the only one that is not a little bug.

You can also do this with pictures instead of words. Make a habit of cutting pictures out of ads, magazines, and whatever else comes your way so that you can play games with the pictures.

Chapter 2 - Island Hopping - Counting

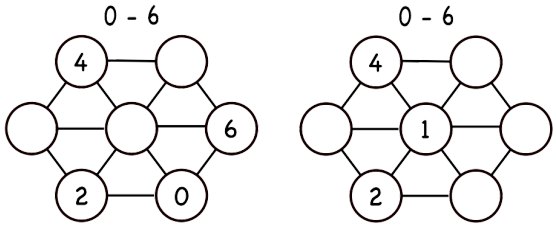
These puzzles have numbered islands (circles) connected by bridges (lines) drawn on paper. The challenge is to find a path that connects the islands in order.



The easiest versions have all the numbers filled in and the numbers go from 1 to the number of islands. You can vary this activity by starting at some number other than 1 and by leaving out some of the numbers.

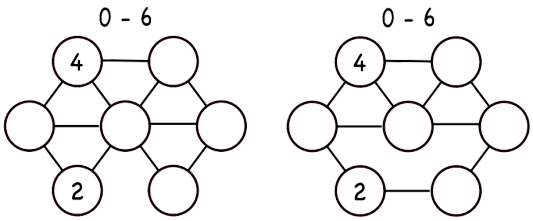
Puzzles with all the numbers are straightforward once your child is confident with counting. Those beginning puzzles are good counting practice and are also good for building up confidence in puzzle solving. The more challenging puzzles are the ones with numbers left out.

Ease your child into these harder puzzles by leaving out just a few numbers and slowly move to leaving out more.



The first of these two puzzles has every other number left out. This makes it relatively easy to fill in the missing numbers. 1 must attach to 0 and 2, and there is only one place for that. 3 must attach to 2 and 4, and with 1 filled in there is only one remaining place for the 3. 5 must go in the remaining place between 4 and 6.

The second puzzle is a little bit harder. 3 must attach to 2 and 4, so there is only one place for it. 5 must attach to 4, so there is only one place for it now. 6 must attach to the 5. Finally, 0 must go in the remaining spot.



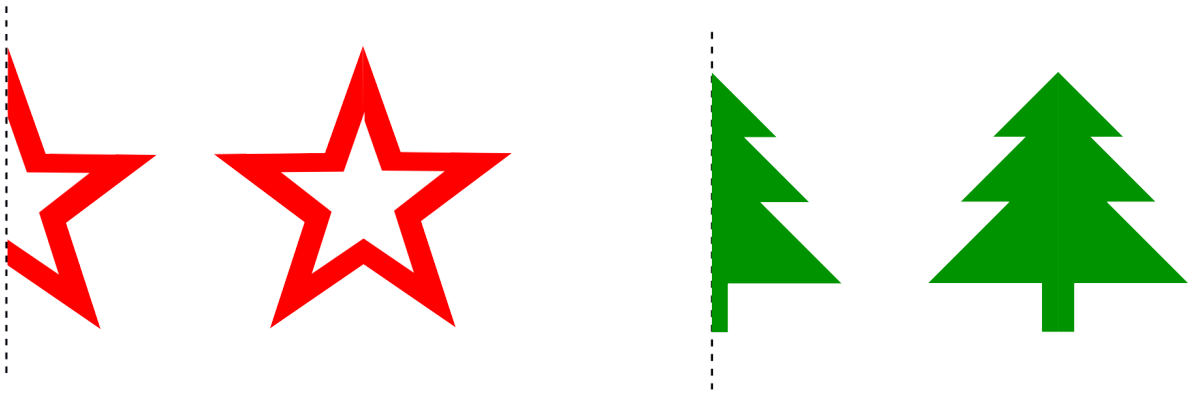
To make the puzzle harder still, we can remove the 1 and play with removing some of the bridges. Have fun with variations and let your child design some of them, too.

Chapter 2 - Cutting Symmetric Shapes

Create designs by folding a piece of paper and cutting the paper while it is folded. This is called Kirigami.

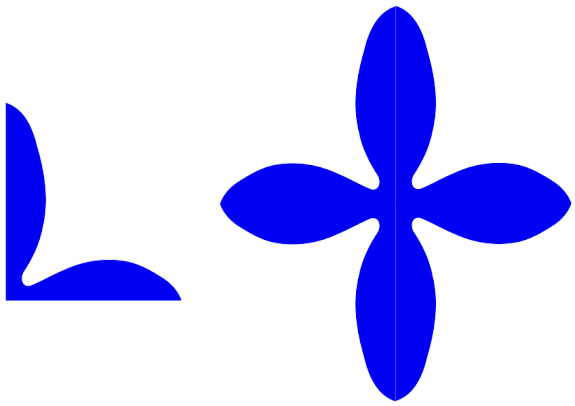
— One Fold —

Folding the paper once and cutting it creates a design with one side the mirror image of the other. Experiment with cutting out faces, lamps, or geometric shapes. The star and tree were produced with a single fold, which is shown on the left, and then the unfolded paper is shown on the right.



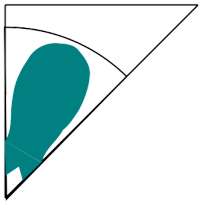
— Two Folds —

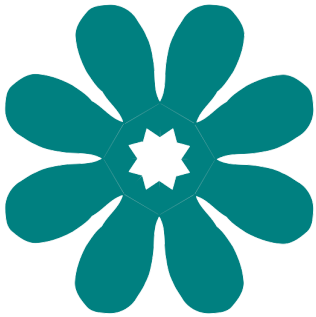
Folding the paper once, and then folding once more in the opposite direction, will produce figures with two lines of mirror images. This makes it easy to create designs such as flowers. The figure on the left is the paper folded twice and cut to leave the blue area, and the figure on the right shows the unfolded paper.



— Three Folds —

Experiment with various folds and cuts. This figure was created by taking a twice folded piece of paper and then folding it once more diagonally through the corner of the previous folds.

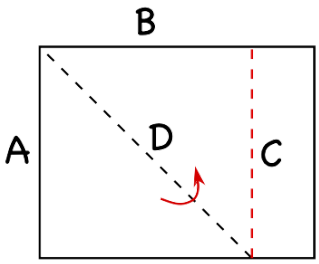


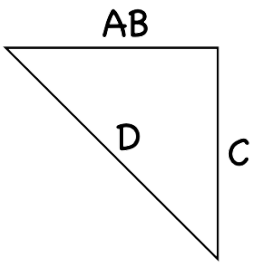


— Snowflakes —

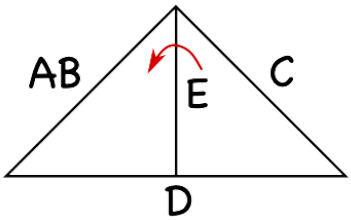
This is a sequence of folds for creating 6-point snowflakes. Though it takes a few steps, don't be put off by them - with a little practice they become quick and easy.

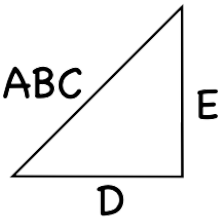
Start by taking a standard sheet of paper and folding at one of the corners so the sides marked A and B meet. Leave the fold in place and cut along the line marked C.



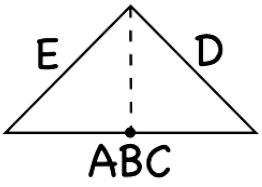


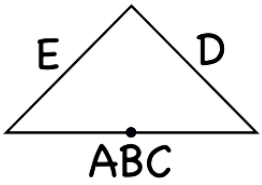
Take the triangle produced and fold it in half so that sides AB and C overlap.



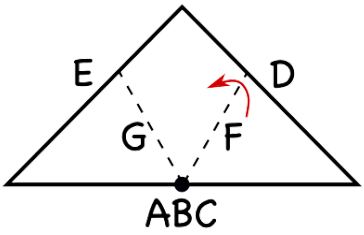


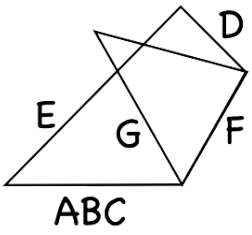
Put a temporary fold in this triangle and use the fold to mark the middle of side ABC. Undo the temporary fold.



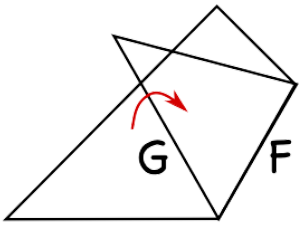


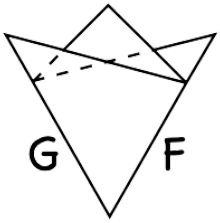
Make a fold over F. When you fold over F, you will be looking to have G placed so that G breaks the angle in half.



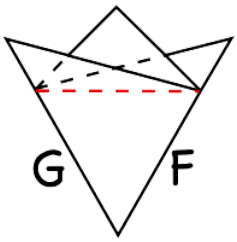


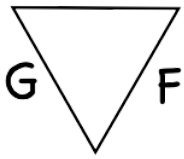
Fold along G - do this fold underneath so that this new folded piece is underneath the other paper.



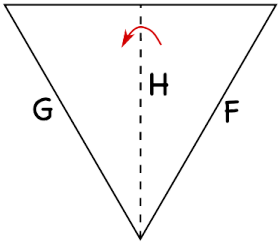


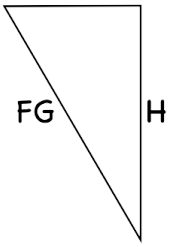
Although not strictly necessary, it is a good idea to slice off the top of this figure. Otherwise, you might be tempted to use the region above the red dotted line without realizing that there isn't paper on all levels.



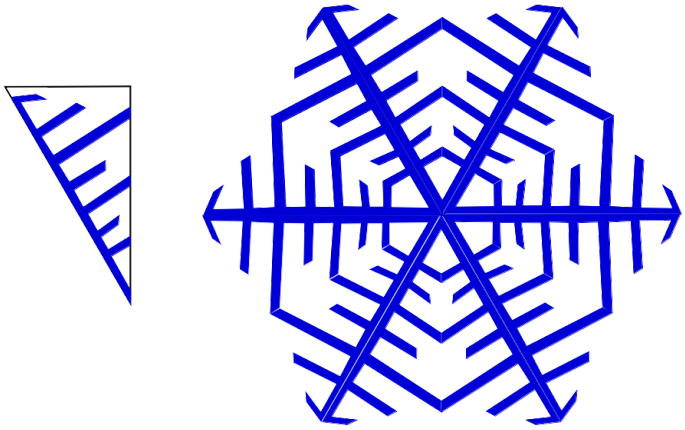


Lastly, fold this triangle in half. At last you are all set to do the cutting of your design!





Have fun experimenting with many different combinations of cuts and colors!



Chapter 2 - Nim with 1 and 2

— The Game Rules —

A target number, say 10, is chosen. Let your child choose whether to go first or second. The total starts at 0. During a turn, a person chooses to add 1 or 2 to the current total. The first person to reach the target wins.

This game can also be played with subtraction. In this version, the starting total starts at the target, which in this example is 10. On a given turn the player chooses whether to subtract 1 or 2. The first person to reach 0 wins.

Another variation is that instead of winning, the player forced to hit or go beyond the target number loses. You can also experiment with what happens if you allow a player to add (or subtract) 1, 2, or 3 for each turn.

— Understanding the Game —

Without analyzing anything, this game is enjoyable to play and it provides good practice with adding or subtracting 1 and 2. We could leave it at that. However, it is also a great example of two problem solving techniques that you can show to your child when they are ready: 1) learning from simpler examples, and 2) looking for patterns.

Any of the versions can be studied this way. Let's look at one: Subtracting starting at 10 and whoever gets 0 wins. The hard part of this game is that 10 is so far from 0. So, let's look at a simpler version. When children are asked to do this, they often suggest starting at 5 or 6 - it seems absurd to them to start at 1, but that is actually what they should do! Often it is best to start as simply as possible - that means starting at 1. If it is your move and the count is 1, you win. Do the next few. If the count is 2, you win. If the count is 3, you must lose - whether you subtract 1 or 2, you will give your opponent a winning position. If the count is 4, you will win because you will subtract 1 and put your opponent in a losing position. Continuing in this way, build up a table of results:

1	2	3	4	5	6	7	8	9	10
W	W	L	W	W	L	W	W	L	W

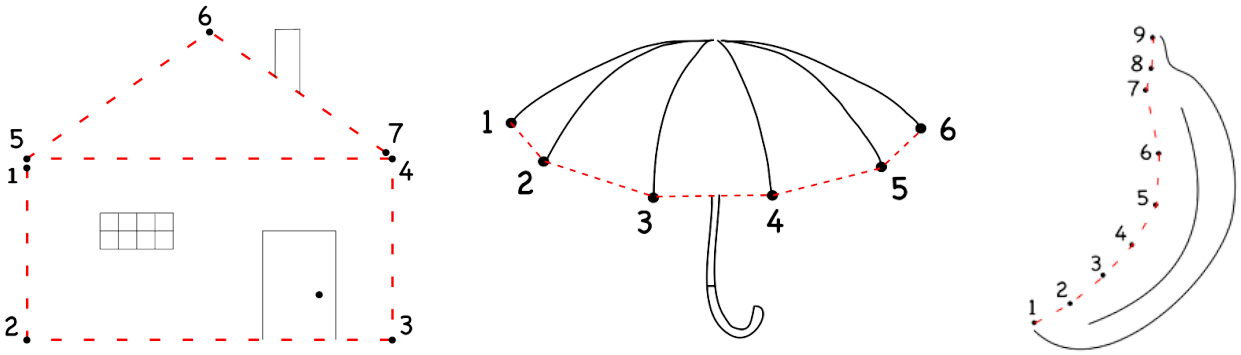
This table makes it obvious that there is a repeating pattern of 3. When starting at 10, you should want to go first and subtract 1. What is satisfying about this is that, once you decide to look at simpler versions of the problem, the analysis is quick and easy - no tricky analysis needed. Now you are a Master at this game and you know what to do starting at any number! Any version of this basic game is just as easy to analyze.

But wait, there is one last question. Why is there a repeating pattern of 3? Once one player is stuck on a losing number which is a multiple of three, every pair of moves after that can be made to add up to 3 - if the losing player subtracts 1 the other player subtracts 2, and if the losing player subtracts 2 the other player subtracts 1.

Chapter 2 - Connect the Dots

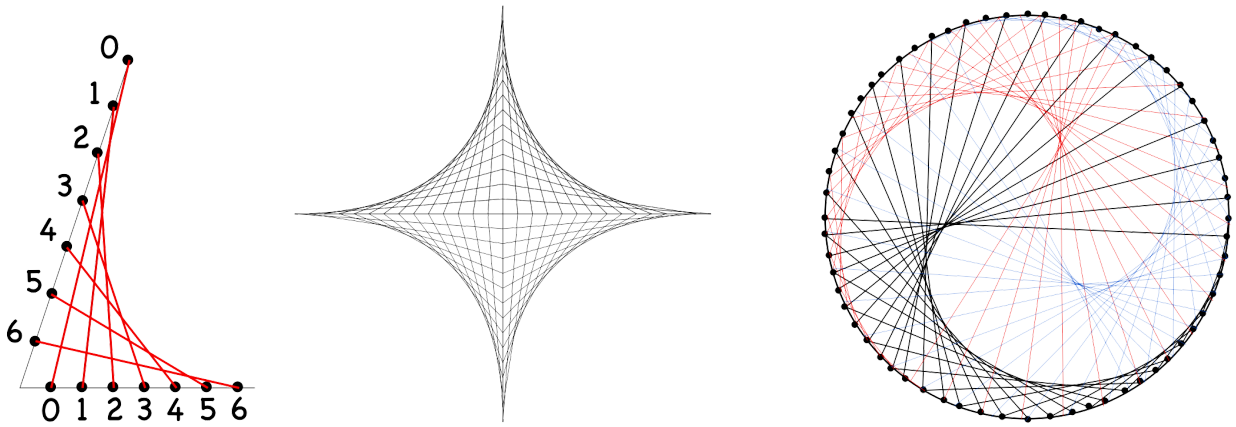
— Making Everyday Scenes With Dots —

Complete fun drawings by connecting numbered dots. One way is to take a simple drawing, say of a house, remove some straight lines and replace them with numbered dots, that when connected in order recreate the original drawing.



— Making Geometric Patterns With Angles —

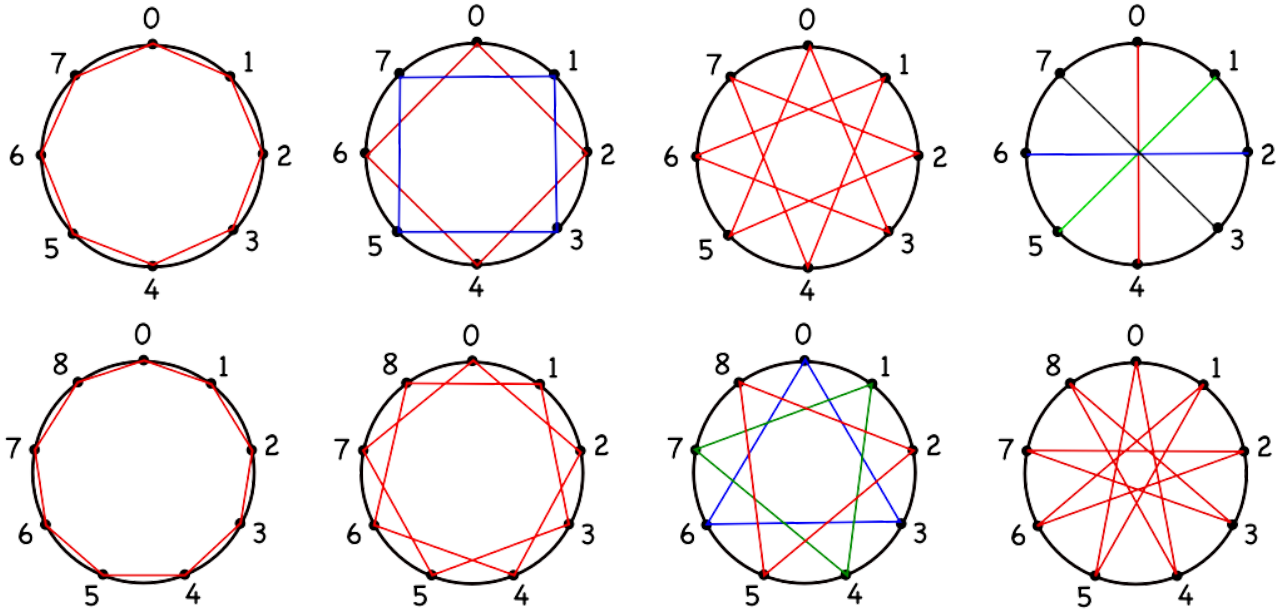
Make abstract drawings by connecting points with the same number along opposite sides of an angle. The numbers may not be needed - if so, feel free to omit them - this will make the finished designs a little less cluttered. You can add variety to these by having your child draw with colors. There are lots of amazing examples of this to be found under the category of String Art on the Internet. This circle drawing was made by moving ahead by one dot on one side of the line segment and ahead by two dots on the other side.



— Making Patterns With Circles —

This is a special case of the last idea. Put some dots, say 8 or 9, evenly spaced on a circle. Your child can play with creating different patterns by connecting the dots in order, or connecting every second dot, or every third dot. To make it easier to do different experiments, use push pins in a piece of cardboard or wood and then use string between the push pins.

If your child is intrigued by the patterns that are produced, you can look at questions such as: For a circle with 8 dots, why is only one string needed to skip by 1, 3, 5, or 7, but 2 or 4 strings are needed for skipping 2, 4, or 6. Similarly, for a circle with 9 dots, why is only one string needed to skip by 1, 2, 4, 5, 7, or 8, but 3 strings are needed for 3 and 6? It's too young to understand the idea that 2, 4, and 6 have a factor in common with 8, and 3 and 6 have a factor in common in 9 - however, seeing the patterns may plant the seeds for later ideas.



Chapter 2 - Shape Sudoku

— Introduction —

This is your child's first math puzzle, and that's pretty cool! It also means you should take it very slowly so your child has lots of success and fun, and very little frustration.

The rules for these 4 by 4 Sudokus are very simple. There are four different types of tokens. There should be one of each kind of token in each row, column, and 2 by 2 corner of the puzzle. Use moveable pieces so that it is easy for your child to experiment in finding solutions.

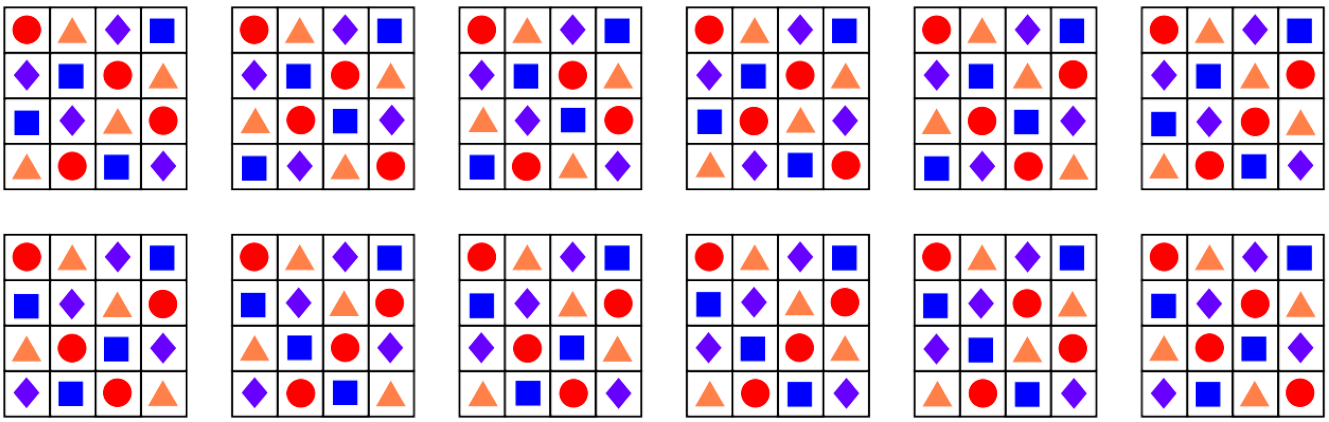
The first puzzles you make should just have one token missing from each row. Once your child understands and figures out those puzzles, you can move to trickier ones, but do not be in a hurry.

The easiest way to create these puzzles is to start with a finished Sudoku and remove some of the tokens. To help you to do that, there are a number of finished Sudokus supplied below. After those finished examples, there is a list of methods you can use to create puzzles from a finished puzzle.

— Finished Sudokus —

Before giving you a collection of finished Sudokus, there is one thing to note. You can take any one of these and create 23 more of them simply by interchanging token types - for example, you can take a finished puzzle and create a "new" one by swapping the circles and triangles and swapping the diamonds and squares.

The examples below are distinctly different from each other and cannot be made from each other by doing interchanges. You can create more examples from these by doing such interchanges if you like.



— Methods for Creating Sudokus From Solved Ones —

Once you have a finished Sudoku, you can use any of the following strategies to create a puzzle that has a unique solution. Generally speaking, the more tokens you remove, the harder the puzzle will be.

- Remove a single token from each row or from each column.
- Remove a single token from each 2 by 2 corner.
- Remove all of one kind of token from the entire puzzle.
- Remove all tokens from one 2 by 2 corner.
- Remove one entire row and one entire column.
- Remove all of one kind of token and one each of the other kinds of tokens.
- Remove all tokens from two 2 by 2 opposite corners.
- Remove all tokens from two 2 by 2 opposite corners and 1 token each from the other two corners.

Of course, these are not the only methods you can use. They are just supplied here as sure-fire general methods that will quickly create puzzles.

Chapter 2 - Number Sudoku with Jigsaws

This is similar to the Shape Sudoku puzzles, only now it uses numbers. If your child is not ready to recognize numerals yet, you can use quantities of dots instead. To avoid erasing, use numbered (or dotted) slips of paper to solve the puzzles.

For a 4 by 4 puzzle, each row and column has the numbers from 1 to 4 once. Also, each marked subregion has the numbers from 1 to 4 once.

Create these puzzles for your child by starting with a completed puzzle with moveable pieces of numbered paper and then removing some of the pieces of paper.

— 4 by 4 Puzzles —

The 4 by 4 puzzles with subregions that are the 2 by 2 corners are exactly the same as the Shape Sudoku puzzles given before. You can go back to that page in these Resources to see solved versions of those puzzles. To create a numbered version of them, replace each colored shape by a number. For example, red circles could be 1, orange triangles 2, purple diamonds 3, and blue squares 4.

1	2	3	4
4	3	1	2
2	1	4	3
3	4	2	1

1	2	3	4
4	1	2	3
3	4	1	2
2	3	4	1

1	2	3	4
4	1	2	3
3	4	1	2
2	3	4	1

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

1	2	3	4
4	3	1	2
3	4	2	1
2	1	4	3

1	2	3	4
4	3	1	2
3	4	2	1
2	1	4	3

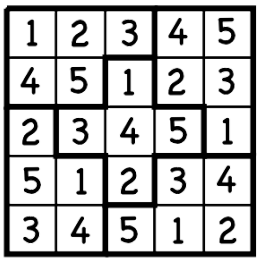
1	2	3	4
4	3	1	2
3	4	2	1
2	1	4	3

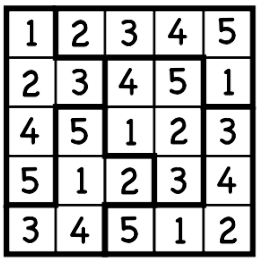
1	2	3	4
4	3	1	2
2	1	4	3
3	4	2	1

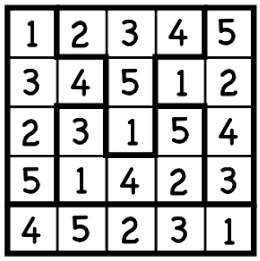
— 5 by 5 Puzzles —

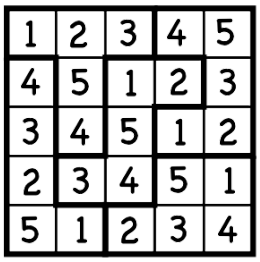
There are too many of these puzzles to come anywhere close to showing all the possible jigsaw geometries. These are here just to suggest what is possible. Your child may enjoy finding different ways of carving up a 5 by 5 square into pieces that have 5 small squares.

Pieces that consist of 5 little squares are called “pentominoes.” Making shapes with pentominoes can be lots of fun. Maybe cut out some pentomino shapes out of stiff thick colored paper and see what designs you can make!



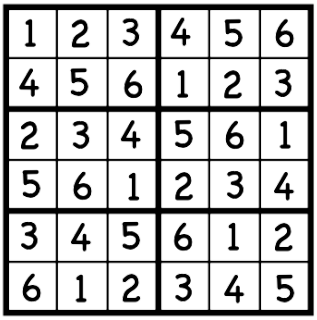


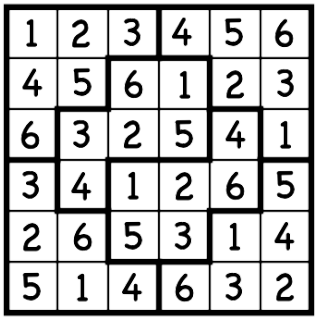


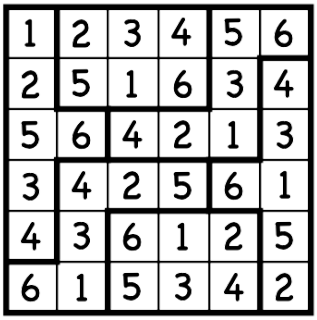


— 6 by 6 Puzzles —

Okay, you are getting the idea. There are a lot of these! Here are just a few 6 by 6 puzzles to give you some ideas for what is possible. As always, play around with your child with these puzzle pieces and numbers. Perhaps design a few of these together.



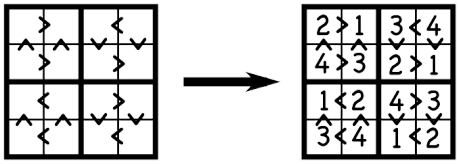




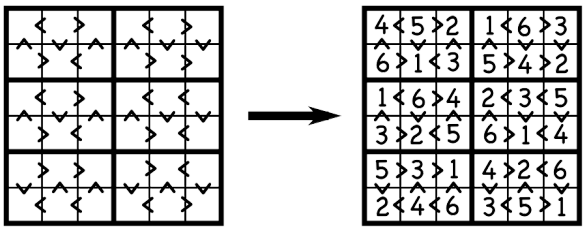
Chapter 2 - Greater Than Sudoku

Greater Than Sudoku puzzles start with the same rules as regular Sudoku - each number appears exactly once in each row, column, and subregion. Additionally, if there is a less than or greater than symbol between two cells, then the numbers in the cells must obey that relationship.

Make these puzzles by using a finished Sudoku puzzle - all the example Number Sudoku Jigsaw puzzles given early in these Resources will be useful in creating these puzzles. Put in greater than and less than signs on a blank grid of the same geometry. If you omit all the numbers and put in all the inequalities (less than or greater than), it is generally fairly easy to solve the puzzle. A useful strategy for your child is to first look for where the smallest and largest numbers should go.



When your child is first learning how to do these puzzles, put in all the inequalities and some of the numbers. Gradually, start omitting more of the numbers and some of the inequalities.



Chapter 2 - Make Me a Liar

Someone makes an absolute statement and the other players attempt to show that the person is lying. This is done by finding an example that breaks the statement.

— Simple Statements That Are Almost Always True —

One type of statement to use is saying that something is always true. Here are some examples with quick discussions of why they are lies.

- All trucks have four wheels. - Large trucks often have 6, 10 or more wheels.
- All rectangles are squares. - Rectangles need not have all their sides the same length.
- All birds can fly. - Ostriches, emus, and kiwis are birds that cannot fly.
- The moon is only visible at night. - The moon is often visible during the day.
- All shapes have straight sides. - A circle does not have a straight side.
- All playgrounds have swing sets. - Some playgrounds don't have swing sets.
- All rooms have chairs. Bedrooms and bathrooms often don't have a chair.

— If - Then Statements That Are Almost Always True —

Another type of statement is of the form "if __, then ___ ." Here are some examples with quick discussions of why they are lies.

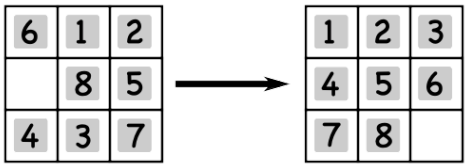
- If today is Monday, then it is a school day. - Some Mondays are holidays and some Mondays occur during the summer.
- If I don't eat for three hours, then I am hungry. - Most people can sleep for more than three hours and not wake up hungry.
- If a person is taller than someone, then they are older. - Children often grow up to be taller than their parents.
- If the sun is shining, it is a warm day. - Winter days can be sunny and cold.
- If someone is late, something bad must have happened to them. - Sometimes people are late through carelessness or reasons beyond their control (traffic, bad weather, car trouble).

Chapter 2 - 15-Sliding Puzzle

— Puzzle Description —

The classic version of this puzzle starts with a 4 x 4 empty grid of squares formed by 5 horizontal and vertical lines. Use a set of 15 pieces of paper the size of the grid squares, and number the pieces of paper from 1 to 15. The puzzle starts by having someone place the pieces of paper on the grid. The object of the puzzle is to get the pieces of paper in order with only the lower right hand corner of the grid empty. To achieve this, a piece of paper can be moved if it is adjacent to the empty square - in which case it can be slid into that space. Depending on how the person sets up the puzzle, the puzzle may or may not be solvable.

A 4 x 4 grid is too hard for a beginner, so start with something smaller. The grid could be as small as 2 x 2 or as big as the child wants. The number of numbered pieces of paper will always be one less than the size of the grid. For example, on a 2 x 3 grid use the cards from 1 to 5.



To create these puzzles, you have two options. The first is to place the squares randomly, in which case you have a 50 / 50 chance of the position being solvable. Alternatively, you can start by placing the pieces of paper in the final position and then making a series of legal moves to move the paper around. When you are all done, you are guaranteed that the puzzle is solvable.

— Solving the Puzzle —

The main reason for a child to play with this puzzle is to have fun moving pieces around until they accidentally solve it and also to practice getting numbers in order. Despite that simple goal, you may start to wonder about deeper ideas in the puzzle.

A frequent theme of problem solving is to learn from simpler problems or examples. So, let's do that.

The smallest example is 2 by 2. For this size, it is clear that the rows will end up being either 1 2; 3 0 or 1 3; 2 0.

The next smallest is 2 by 3. Start this by getting the 1 and 4 in the left column. Once this is done, your puzzle will look like 1 _ _; 4 _ _ . Finish off the last four squares as you would the 2 by 2 case.

The 2 by 4 puzzle is done similarly. Start by putting 1 and 5 in the left column. Next, put the 2 and 6 in the second left column without disturbing the 1 and 4. Finally finish off the last 2 by 2.

At this point, the pattern for attacking puzzles with 2 rows is clear. What to do with more than 2 rows? Suppose you have 3 rows. Start the solution by getting the top row correctly laid out. After that, leave the top row undisturbed and use your ability to solve a puzzle with 2 rows.

Similarly, if there are 4 rows, do the top row first, the second row next (without disturbing the top row), and finish off the last 2 rows as before.

— Is This Puzzle Solvable? —

Okay, you have a simple method for solving the puzzle. The next question is: How can I just look at the puzzle and know whether it is solvable or not?

To make describing the answer as simple as possible, do a few quick moves, if needed, to place the empty square in the bottom row. Next, make a list of the rows in one long list - the first row is listed first, the second row is listed second, and so on with the last row listed last. Omit the empty square when you list the last row.

Take this long list and count the number of inversions in it. When a number earlier in the list is larger than a number later in the list, this is called an inversion. If the number of inversions is an even number, then the puzzle is solvable. If it is an odd number, it is not.

As an example, take the 3 by 3 puzzle at the start of this discussion. Start by moving the 4 up to the second row. Then the list is: 6 1 2 4 8 5 3 7. There are 10 inversions in this list: 6 1, 6 2, 6 4, 6 5, 6 3, 4 3, 8 5, 8 3, 8 7, and 5 3. There are an even number of inversions, so the puzzle is solvable.

Why does this rule work? I won't drag you through a detailed analysis. The key idea is to keep track of the number of inversions every time you make a move. It turns out that, if you adjust for the hole being in the last row, the number of inversions must always change by an even number after any move. Consequently, if the number of inversions starts as an odd number, it can never get down to 0 inversions.