Early Family Math

Stage 4 – I Can Count to 20!

Prerequisite: Can count to 20 comfortably with a good sense of those quantities. Comfortable adding and subtracting small numbers and understands number bonds and fact families.

Where You've Been

Your child has come so far! They can count up and down between 0 and 20 and understand what those quantities mean. Your child is comfortable "counting on" or "counting down" starting at any number in between 0 and 20. There is also a budding understanding of place value and how that relates to expanded form.

Calculational abilities have also grown! Beyond adding and subtracting small numbers, your child has a sense of the number relationships of number bonds and fact families. The ideas of adding and subtracting 1 or 2, adding twins, and near twins, are ready to be the foundation for learning all addition and subtraction facts. Your child has also picked up skip counting by 2, doubling, halving, and even and odd numbers – all of which set the stage for learning multiplication and division.

New Ideas in this Stage

- **Counting to 100** – While it's fun to hear your child count to 100, keep this practice grounded in understanding the numbers – place value and counting backwards will help a lot with this.
- **Expanded Form and Place Value** – The earlier practice with this will be reinforced here. Understanding tens and ones, and using expanded form (such as 37 = 30 + 7), is essential to understanding numbers.
- **Comparing double-digit numbers** – Understanding place value will make this easy.
- **All single-digit adding and subtracting** – This is first done with manipulatives, especially fingers. Then, using number relationships, it will be extended to mental arithmetic.
- **Compensation for Addition and Subtraction** – Adjusting addition and subtraction problems to make them easier helps with mental arithmetic, and teaches the structure of numbers.
- **Skip Counting** – Skipping up and down by single-digit numbers starting anywhere is great for doing mental addition and subtraction, and helps with multiplication and division.
- **Beginning multiplying** – Your child already knows how to multiply by 2. This will be extended, in part using skip counting, to all single-digit numbers.
- **Games, Puzzles, Problem Solving and Investigations** – The games and puzzles will be more challenging in keeping with your child's growing sophistication and abilities. Keep these activities fun and playful - enjoyment is the most important thing for your child's long term mathematical success. Kindle sparks of joy when your child discovers or experiences particularly beautiful or satisfying mathematical patterns.

FINGER MATH – SINGLE-DIGIT ADDITION

Prerequisite: Comfort adding and subtracting small single-digit numbers with manipulatives

Introduction

These teaching activities give simple, reliable methods for your child to understand and perform basic addition and subtraction.

Finger Addition with Compensation

Activity

Use compensation for sums between 11 and 18 to make them easier.

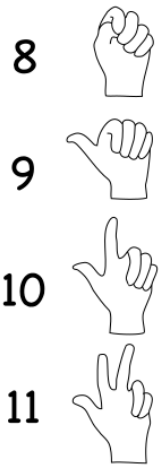
Giving away fingers: Suppose you're adding 7 + 8. One person puts up 7 fingers and the other person puts up 8 fingers. Then, one person gives away as many fingers as are needed to put up 10 fingers on the other person's hands. In this example, 7 + 8 could turn into 5 + 10 (giving away 2) or 10 + 5 (giving away 3).

Magic: Be dramatic and give away the fingers by having one person's hand bang into the other person's hand and having the fingers transfer "magically."

Finger Addition the Easy Way

Activity

Counting on: Use the idea of 'counting on' to make addition easier. Counting on refers to doing addition by starting at one of the numbers, rather than starting at 0 and counting out both of the numbers.



Let's use adding 8 + 3 as an example. It is easier if you pick the larger number being added as the starting point for the 'counting on.'

Have your child make a closed fist and say '8.' Then, lift one more finger each time as your child counts out loud '9, 10, 11.' When 3 fingers are raised, you are done adding 3, and the counting stops. At that point, you have that 8 plus 3, which is 11.

With practice and further number relationship ideas, these math facts will become automatic for your child. However, there is no hurry for memorization, and it can wait until more experience with the quantities and relationships between the numbers has been gained.

FINGER MATH – SINGLE-DIGIT SUBTRACT

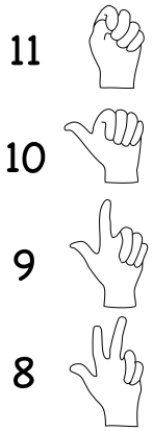
Prerequisite: Comfort adding and subtracting small single-digit numbers with manipulatives

Finger Subtraction the Easy Way

Activity

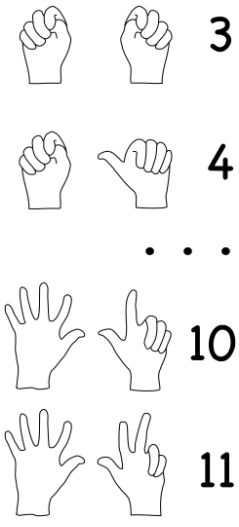
Two methods: Subtraction can be thought of as 'take away' or 'difference.' Both models are essential to a complete understanding of subtraction. Have your child practice both ways of thinking of subtraction using these finger subtraction methods.

We'll use 11 - 3 for our examples.



Take Away method: Start with a closed fist and say "11." Then, lifting one more finger each time, count "10, 9, 8." Each time you lift a finger and count down, you are taking one more away from the original number.

When your child sees 3 fingers raised, the counting stops. They now see that taking 3 away from 11 leaves 8.



Difference method: This uses counting on, much as we did for Easy Way Finger Addition. What we are doing is finding out which number we need to add to 3 to get 11.

Have your child make a closed fist and say "3." Then, lifting one more finger each time, count "4, 5, 6, 7, 8, 9, 10, 11." When your child says 11, there are 8 fingers raised. That shows that the difference between 3 and 11 is 8!

SINGLE-DIGIT ADDITION

Prerequisite: Comfort adding and subtracting single-digit numbers with manipulatives

Pig

Game

How to play: During a turn, roll a die as many times as you want. When a roll is not 1, you add that roll to your turn's total. If it is 1, you lose everything for that turn and the turn is over. A player may choose to stop before rolling a 1, keep the turn's points so far, and add them to the player's running total.

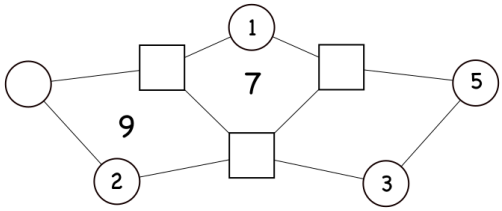
How to win: The first player to reach the target number, say 30, wins.

Variations

A two dice variation has these rules: If neither die shows a 1, the sum is added to the turn's running total. If exactly one of the dice shows a 1, then nothing more is added to the running total and the turn ends. If two 1's are rolled, the turn's total becomes 0 and the turn ends.

Enclosed Sums

Puzzle



The setup: These puzzles have shapes connected by lines. Each enclosed region, such as where the 9 or the 7 are in this example, has a number that is the sum of the shapes that border it.

While circles may have any value, a non-circle must have the same value as any other figure of the same shape.

The challenge: Fill in the missing numbers in the shapes and the inside of the regions.

How to create: Create these puzzles by making a diagram of circles and maybe some squares. Next, fill in all the figures with numbers and fill in the bounded regions with the sum of the figures that surround them. Finally, remove some of the numbers.

NUMBER BONDS AND FACT FAMILIES

Prerequisite: Comfort adding and subtracting single-digit numbers with manipulatives

War – Addition and Subtraction

The setup: Evenly split a shuffled deck of playing cards with the face cards removed. If you like, use dominoes instead.

How to play: Both players turn over their top two cards and add them. The player with the larger sum wins all four cards. If the sums are equal, the next two pairs of cards are added and the winner gets all eight cards. Play this with a single pass through the deck or multiple passes.

How to win: The winner is the player who has the most cards.

Variations

For variety, play this using the difference of the two cards. Or, you can add three cards at a time. Another option is to assign one person to be Odd and the other Even. For this, each player turns over a card and the evenness or oddness of the sum determines who gets the cards.

Target Gin Rummy

The setup: Agree on a target sum, say 10. Remove the face cards and deal seven cards to each player. The remaining cards become a draw pile, and its top card is flipped over to start the discard pile. The goal is to hold seven cards that are broken into separate groups of one or more cards that add up to the target.

How to play: During a turn, the player has the choice of picking up the top card of the discard pile or the unseen card at the top of the draw pile. That player then discards a card.

How to win: When a player successfully fills their whole hand, the player lays down the hand and says “Gin!”

Variations

Taking the difference with pairs of cards can be used instead of addition. In that case, deal an even number of cards to each player.

MENTAL MATH – SINGLE-DIGIT ADDITION

Prerequisite: Ability to do single-digit addition and subtraction using manipulatives

Introduction

These teaching activities give techniques for learning basic addition and subtraction.

Flash Cards

The ease of practicing math facts using flash cards makes them tempting. However, they are often abused by well-meaning helpers and can contribute to math hatred. Beyond the psychological damage that occurs with overly-enthusiastic drill, using flash cards misses out on learning important relationships between numbers. Feel free to use flash cards to give focused practice for a small handful of facts, but please keep the practice limited and relaxed.

This page and the next have methods that practice structural insights that are useful and far more interesting to your child, and they should be used until the facts become automatic.

Reviewing Addition Skills

To do the activities on this page your child should know how to mentally do the following:

- Add and subtract 0, 1, 2 (and perhaps 3)
- Do adding twins and near twins
- Know the number bonds for 10
- Add 10 to single-digit numbers.

If your child is weak with any of these skills, this is the time to practice those skills some more.

Addition Compensation

Compensation is a powerful tool for making mental math easier. When adding two numbers, you can get the same sum by shifting over part of one number to the other. Adding 8 or 9 is easy using compensation. For example, add 6 + 9 by shifting 1 from the 6 to the 9, which gives 5 + 10. Similarly, 4 + 8 becomes 2 + 10.

Use compensation from twins and near twins to make all of the remaining math facts easier: 3 + 5, 3 + 6, 4 + 7, and 5 + 7. For example, using compensation, 5 + 7 is the same as 6 + 6.

More than one way: Some math facts can be done several ways. Challenge your child to find more than one way to do a problem. For example, 5 + 7 can become 6 + 6, but it can also become 2 + 10. This kind of math play will lead to lasting insights.

MENTAL MATH – SINGLE-DIGIT SUBTRACT

Prerequisite: Ability to do single-digit addition and subtraction using manipulatives

Reviewing Subtraction Skills

Before starting these subtraction activities, practice any of the following skills that happen to be weak for your child:

- Add and subtract 0, 1, 2 (and perhaps 3)
- Subtract numbers 1 or 2 apart
- Know the number bonds for 10 and how they make subtracting from 10 easy
- Subtract 10 from numbers from 11 to 19

Using 10 as an Intermediate Stop

For problems with numbers larger than 10, such as 13 - 8, break them into two differences. Make 10 an intermediate stop when going between the two numbers. The distance from 13 to 8 is the distance from 13 to 10 plus the distance from 10 to 8. Using this, 13 - 8 becomes (13 - 10) + (10 - 8) = 3 + 2 = 5.

This simplifies the mental load considerably by breaking these subtractions into two manageable parts. Subtracting 10 from a number between 10 and 20 is very straightforward. Learning how to subtract numbers from 10 is a matter of learning the number bonds for 10.

Subtraction Compensation

Compensation for subtraction means adding or subtracting the same amount to both numbers. By adding or subtracting the same thing, the distance or difference between the two numbers is maintained.

Use compensation on 13 - 8 by adding 2 to both numbers to turn the problem into 15 - 10. Notice how much easier the problem became using this minor adjustment!

Single-digit problems can also be done this way. For example, 3 can be added to both numbers in 7 - 3 to make it 10 - 6, which is 4.

SINGLE-DIGIT ADDITION

Prerequisite: Comfort adding and subtracting single-digit numbers with manipulatives

Don't Go Over

Game

The setup: Use 5 dice and 4 rolls.

How to play: On the first roll, choose to save from 0 to 5 of the dice. Once a die is saved it cannot be changed. Similarly with the remaining dice on rolls two and three. On the final roll, all dice are saved. Any score less than or equal to 20 counts, any score over 20 gives the player 0.

How to win: You can play one or more rounds. The highest total score wins.

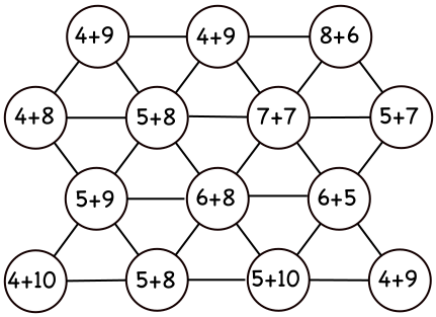
Variations

The target score of 20, the number of dice, and the number of rolls, can all be changed to suit younger or older players. For example, play this with a target of 12 and 3 dice.

Island Hopping – Compensation

Puzzle

These puzzles show how using compensation creates easier addition problems.



The challenge: Find a path that connects all the islands that have the same answer. Two islands can only connect if their problem's numbers differ by 1 – with one number increased by one and the other decreased by one.

For example, '5 + 8' would connect with '6 + 7.' Only some of the islands will be on the path. In this example, the path starts in the lower left and ends in the upper right.

How to create: Make these puzzles by starting with about ten empty circles with some connections. Identify a path from one edge of the islands to the other. Along that path, put in problems whose numbers differ from each other by one. In the nearby islands, put problems with small changes that have different answers.

GAMES FOR ADDING AND SUBTRACTING

Prerequisite: Ability to do single-digit addition and subtraction using manipulatives

Part-Whole Triangles

Game

The setup: Use number cards 1 to 13. To start, each player is dealt 6 cards face up. There is a draw pile with one card turned over to start a discard pile.

How to play: During a turn, a player takes the top discard card or the unknown top draw pile card. The chosen card replaces a card they already have, and the replaced card is discarded.

How to win: The first player to create a 6-card pyramid where each card is the sum of the two cards below it wins.

Variations

Use smaller or larger ranges of cards to match the skills of younger or older players.

Cards to a Target

Game

The setup: Start by laying out all the playing cards from 1 to 5 in a 4 by 5 grid. Start the running total at 0 and choose a target number, say 25.

How to play: Players take turns turning over one number and adding it to the running total. The last player to pick a number that does not run over the target number wins.

Variations

Replace 1 to 5 with any five numbers you want to practice. To practice subtraction, start at the target number, subtract the selected numbers, and don't allow going below 0.

Get Out of My House – Addition and Subtraction

Game

The setup: Use a deck of cards with numbers from 1 (Ace) to 10. On a shared piece of paper, draw 20 boxes or simple houses numbered from 0 to 19. Each player has 7 tokens distinct from the other player's 7 tokens.

How to play: During a turn, a player selects two random cards, and can choose to add, subtract, or multiply them to put their token in a house with fewer than three of the opponent's tokens. If the house contains one or two of the opponent's tokens, those tokens are given back to the opponent and the player says "Get out of my house."

How to win: The first player to put all their tokens in houses wins.

GAMES FOR ADDING AND SUBTRACTING

Prerequisite: Ability to do single-digit addition and subtraction using manipulatives

Combo Dominoes

Game

The setup: Use a set of dominoes that either go from 1 to 6 or 1 to 9. Each player starts with 5 random dominoes without letting the other player see them.

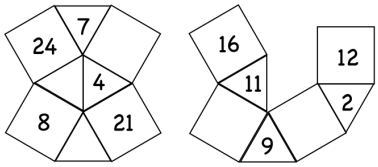
How to play: To start, a random domino is placed face up in the middle. After that, a player must match the top domino in the middle. Matching means that the two numbers on the top domino can be combined with any operation – add, subtract, or even multiply if you want – to make the same result as some, possibly different, operation acting on the two numbers of one of your dominoes.

For example, if [1,5] is on top, then it matches [2,4] because 1+5 = 2+4, and it also matches [2,2] because 5-1 = 2x2. The matching domino is placed on top of the previous top. If you can't make a match, you must pick up a new domino from the pile.

How to win: The first player to get rid of all their dominoes wins.

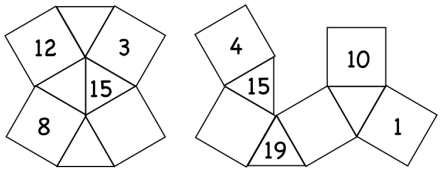
DiffTriangles and SumTriangles

Puzzle



DiffTriangles: These puzzles have triangles and squares that share sides. A triangle always has exactly two squares on its sides – the remaining side has either a triangle or is empty. A triangle's number is the difference of the two adjoining squares.

SumTriangles: These puzzles use addition in place of subtraction. The value of a triangle is the sum of its two or three square neighbors.



The challenge: Fill in the missing numbers to make each type of puzzle work.

How to create: Making puzzles without loops is easy. Draw an alternating sequence of squares and triangles. Then put in numbers starting at one end working your way to the far end. When you are done, remove some of the numbers. Look at the Bonus Material for ideas on how to make these puzzles with loops.

SKIP COUNTING

Prerequisite: Comfort adding and subtracting single-digit numbers, beginning place value

Skip Counting

Activity

Your child has already practiced skip counting by 2's, 5's, and 10's between 0 and 20. Start practicing skip counting by any number, starting at any number, going in either direction.

Skip counting up **and down** helps with all arithmetic operations, as well as place value (when skipping by 5's or 10's). The tricky part in skip counting is when the tens digit changes, so focus on that. This is a handy activity to do when you are traveling or have some idle time.

Short Steps to a Target

Game

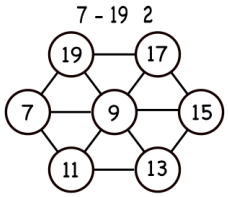
How to play: Create a 2-digit starting number using two playing cards from 1 to 9 - the first card gives the tens place and the other the ones place. Starting at that number, use 5 jumps each of which is either by 1 or 10. The goal is to get as close to 50 as possible, and the score is the difference from 50. The lowest total score after several rounds wins.

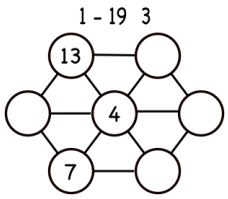
100 chart: Young players benefit from referring to a 100 chart. Using that chart will also emphasize place value as they go up or down by 10.

Variations: You can vary this game by allowing steps of 1, 2 or 10, or steps of 1, 2, 5, or 10. Also, use target numbers other than 50 sometimes.

Island Hopping – Skip Counting

Puzzle





The islands (circles) are connected by bridges (lines), with connections made by skip counting. Some islands have numbers, and others start blank. Above the puzzle is the starting number, ending number, and skip amount.

The challenge: Fill in the missing numbers and find the path.

You can also place the numbers and blanks on the floor to make a stepping puzzle.

As with the Skip Counting activity, create puzzles to practice going forward or backward starting at a variety of numbers, not just numbers that are a multiple of the skip amount.

How to create: Create these puzzles by making the islands first, filling in the skip counting numbers, connecting those islands in the correct order, and then adding some additional connections to help make a puzzle out of it. In the version you give your child, remove some numbers leaving enough of the numbers so that it can still be figured out.

DO SUM PLAY

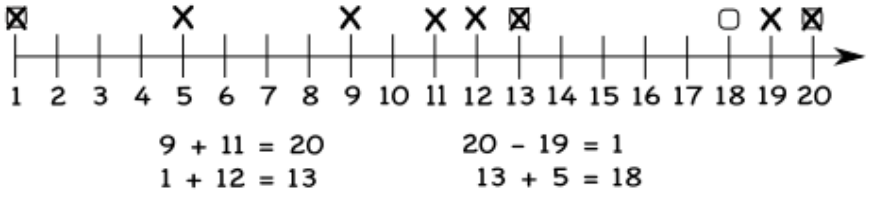
Prerequisite: Comfort adding and subtracting single-digit numbers

Pairing Down

Game

The setup: Start with a number line that goes from 1 to some number, say 20.

How to play: During a turn, choose two numbers and a result, none of which have been crossed out, and write down an addition or subtraction equation that involves those numbers. The two numbers in the equation are crossed off, and the result is circled. The next player must use the result as one of the two numbers.



How to win: If it is played competitively, the winner is the last player with a legal move. It can also be played cooperatively to see how few numbers are left untouched.

Fix It

Puzzle

Target = 8

6	3	5	2
2	1	4	5
3	4	1	3
6	4	2	5

This starts with a completely filled 4 by 4 grid of numbers with a target sum. An alternative version uses individual target sums for each row and column.

The challenge: Find entries to remove so that the sum of the remaining numbers in every row and column is the target.

How to create: Make these puzzles by putting in pairs or triples of numbers that sum to the target sum. Then fill in the remaining spaces with decoy numbers.

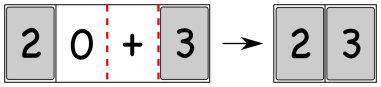
EXPANDED FORM AND PLACE VALUE

Prerequisite: Comfort adding and subtracting single-digit numbers, beginning place value

Folding Expanded Form

Activity

This gives a physical way to see how two-digit numbers are formed from tens and ones.



Take a piece of paper that exactly fits four Number Cards side by side. Mark the paper with: Space - "0" - "+" - Space. Have vertical folds on both sides of the "+" sign. Paper clip numbers to the two spaces. If you use 2 and 3, folded up this looks like 23, but unfolded this becomes 20 + 3.

I'm Thinking of a Number

Activity

One player thinks of a number between 0 and 99. The other player figures out the number by asking questions about the tens and ones digits.

Suppose the number is 23. The player could ask if the tens digit is bigger than or equal to the ones digit – it isn't for 23. The player could ask if the sum of the two digits is less than 8 – it is for 23. The player could then ask if twice the tens digit is bigger than the ones digit – it is. At this point, the number must be 23 or 34. Asking if the sum of the digits is less than 6 finishes things off.

The types of questions can be anything the players agree to, but it is best if the questions involve the ones and tens digits.

Island Hopping by Ones and Tens

Puzzle

A rectangular grid of numbers is given with some numbers filled in. Fill in the remaining numbers so that any two numbers that share a side only differ in a single place, and the difference of the digits in that place is 1 (including going between 0 and 9). No number may be used more than once. Using a 100-Chart may be helpful for beginning solvers.

57	67	66	56
5	4	94	95

33	23	13
32	22	12

How to create: Take an empty grid and fill it with numbers, with no number repeated. Next, remove some of the numbers. In the example, the red numbers are the missing ones.

DO SUM PLAY

Prerequisite: Comfort adding and subtracting single-digit numbers

Math Tic Tac Toe

The setup: Use a Tic-Tac-Toe board and tokens with the numbers from 1 to 9 on them. One player has the odd numbers and the other the even ones.

How to play: Players take turns placing a token, with the Odd player going first. The first player to complete 3 in a row whose sum is 15 wins. One variation is to keep going, fill all the squares, and see which player made the most 15's.

A related game is to have an attacker and a defender. The attacker goes first (the first move cannot be a 5 in the center) and tries to get 15, and the defender tries to stop the attacker.

Memory Challenge Revisited

The setup: Deal a grid of cards face down. A 4 by 4 grid is a good size.

How to play: The players take turns turning over two cards. If the cards 'match,' the player keeps the cards, two more cards are dealt into the empty spaces, and the player gets another turn. If the cards don't match, the cards are turned back over and the player's turn ends.

How to win: At the end of the game, the player with the most cards wins.

Variations

Here are ideas for how cards can match:

- *Use a target sum. Two cards match if their sum is the target.*
- *Use a target difference. Two cards match if their difference is the target.*
- *Use cards with addition or subtraction problems together with cards that have the answers - cards match if the problem matches the answer.*
- *Use expanded form. Select 20 numbers from 0 to 99. Write those numbers on pieces of paper. Also, write each one in expanded form on a piece of paper. For example, create 50 + 3 for 53, 30 + 0 for 30, and 0 + 7 for 7. Shuffle those two decks of 20 cards together. Two cards match when a regular number is paired with its expanded form.*

COMPARING DOUBLE-DIGIT NUMBERS

Prerequisite: Comfort adding and subtracting single-digit numbers, beginning place value

Get Closest

The setup: Write the numbers 5, 10, 25, and 50 vertically on a piece of paper. Put a single blank space on each side of the 5, and two blank spaces on each side of the other numbers. One player fills in the blanks on the left side and the other fills the other side. Each player also has one extra blank to use once with a number to ignore.

How to play: Play with a deck of Number Cards from 0 to 9. Randomly pick a card from the deck, and put it back in after it is used. Both players must use that number somewhere in the spaces that haven't been filled in yet. Once all the spaces are filled, the player's values are compared to each of the target numbers. Whichever player gets closest to each target number gets a point, with both players getting a point if they are equally close.

How to win: Whoever has the most points wins.

Variations

Vary this game by having a different set of target numbers. You can also score the game by summing all the errors for each player – the player with the smaller score wins.

War – Double-Digit Comparison

The setup: Shuffle a deck of playing cards with the face cards and tens removed, and split it evenly between two players.

How to play: Each player turns over two cards and puts them side by side to form a two-digit number. The player with the larger number keeps all four cards. If there is a tie, each player turns over two more cards with the winner getting all eight cards.

How to win: After one or more passes through the cards, the player with the most cards wins.

COMPARING DOUBLE-DIGIT NUMBERS

Prerequisite: Comfort adding and subtracting single-digit numbers, beginning place value

Math Blackjack

Game

The setup: Traditionally, the target number is 21, but for a young child use a smaller number such as 12. Adjust the contents of the playing cards for your child. For example, for a very young child this might be the cards 1 to 4 in the four suits.

How to play: Two cards are dealt to each player - one is face up and one is face down (the receiving player is the only one to look at the face down card). During a turn, the player has the option of asking for one more card until the player decides to stop. After every player has had a turn, the players compare the sum of their cards.

How to win: The player with the sum closest to the target without going over wins.

Fill in the Blanks – Comparison

Game

The setup: Shuffle a deck of playing cards with the numbers 1 to 9.

How to play: Deal two cards to each player face down. Then, each player turns over one card and decides whether that card will be the tens or ones card. After deciding, each player's remaining card is turned over and is used to fill the remaining place.

How to win: The player with the larger number wins.

Variations

- *Play that the smaller number wins.*
- *Decide whether it is more dramatic to show the cards as they are turned over, or wait until all the decisions are made and the final numbers are formed.*
- *Deal three cards to each player and let the players choose which one to put aside.*
- *To practice a bit of addition, as well as making the decisions trickier, draw three cards to turn over one at a time to form a two-digit number and a single-digit number. The goal is to create the largest sum of the two numbers.*

MORE GAMES FOR ADD/SUB

Prerequisite: Comfort adding and subtracting single-digit numbers

Target O's and X's

The setup: Use a deck with the picture cards removed. Fill a tic-tac-toe board with randomly generated numbers from 1 to 20. Use a larger range if you want to include multiplication.

How to play: Deal 6 cards to each player and then flip them all over at the same time. For the first play, it is a race among the two players – the first player to combine two or more of their cards to match one of the squares gets to put an X there and replaces the cards they used. After that, the players take turns putting an X or an O in a chosen square whose value they can match – the cards used for the match are replaced by drawing new ones. If no match can be made, they lose their turn and can choose two of their cards to replace with new ones.
How to win: The first player to get 3 in a row wins.

Terminator 2

The setup: Use three dice and a board with three rows of five squares numbered from 1 to 15.

How to play: A player rolls the dice and uses addition and subtraction to combine the three numbers to match one of the numbers on the board. The matched number is crossed out and claimed. If a player can't find a match, the other player gets a chance to use the numbers and claim the result - in any event, the other player gets the next turn.

How to win: The winner is the one with the most claimed numbers after a fixed number of turns.

Variations

A smaller version would use two dice with the numbers from 1 to 10, and a larger version would use 4 dice and the numbers from 1 to 20.

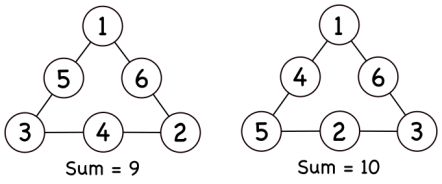
SOLITAIRE SHAPE PUZZLES

Prerequisite: Comfort adding single-digit numbers

Magic Triangles

Puzzle

The challenge: Place the numbers 1 to 6 in a triangle of six circles with three circles on a side, so each side has the same sum.

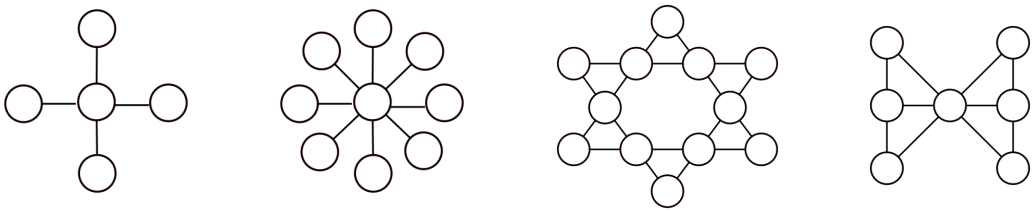


There are really two challenges – find which sums work and how to get those sums. Let your child play with this to figure out which sums work, but if frustration wins out, the sums are 9, 10, 11, and 12.

If your child enjoys this puzzle, this can be done for larger triangles as well. For a triangle with nine circles with four circles on a side, the possible sums are 17, 19, 20, 21, and 23.

Magic Designs

Puzzle



The challenge: Put the numbers in the circles so every straight line of connected circles has the same sum.

From left to right, the diagrams above are for puzzles (2), (4), (8), and (9). The answers are in the Stage 4 Bonus Material file.

1. The numbers 1 to 4 – a '+' shape with no circles in common.
2. The numbers 1 to 5 – a '+' shape with one circle in common in the middle.
3. The numbers 1 to 7 – a 'star' shape with lines of 3 circles; a circle in common in the middle.
4. The numbers 1 to 9 – a 'star' shape with lines of 3 circles; a circle in common in the middle.
5. The numbers 1 to 5 – an 'L' shape with one common circle in the corner.
6. The numbers 1 to 8 – a '+' sign with no circles in common.
7. The numbers 1 to 9 – a '+' sign with one circle in common in the middle.
8. The numbers 1 to 12 – a 'star' shape, with 6 directions of lines of 4 circles.
9. The numbers 1 to 7 – an 'H' shape - 3 vertically on the left, 1 in the center, 3 vertically on the right. The five possible lines of 3 are connected. Hint: The sum is 12.

MORE MENTAL MATH WITH 10'S

Prerequisite: Comfort with single-digit adding and subtracting, number bonds

Grabbing 10's

Activity

Turn long addition problems into simpler problems by grouping numbers that add up to 10. Instead of adding 3 + 8 + 9 + 4 + 7 + 6 + 2 from left to right, rearrange the terms to put the number bonds for 10 together. This example becomes (3 + 7) + (8 + 2) + (4 + 6) + 9, which is 10 + 10 + 10 + 9 = 39.

Once this is easy, include trickier problems such as 4 + 8 + 9 + 5 + 3, which can be rearranged as (8 + 9 + 3) + 4 + 5 = 20 + 9 = 29. Make simplifying expressions a game with your child.

Cover Up

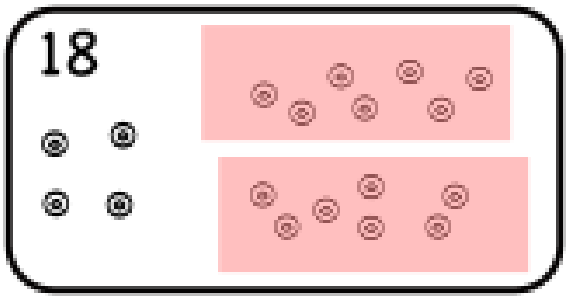
Activity

The setup: This activity combines the approaches of 'What's Missing?' from Stage 2 and 'Shape Sums' of Stage 3. Take a collection of small objects, count them, and spread them out on a surface. Use pieces of paper, cloth, or bowls to cover over one or more groups of the objects. Use the same color paper when covering groups of the same size.

The challenge: Find out how many objects are in each covered group.

Four steps

- *Count the number you can see and compare to the total.*
- *Find one way to solve it.*
- *Find other ways to solve it.*
- *Verify your answer(s).*



Example: As pictured above, suppose you have 18 items and you cover 14 of them with two red pieces of paper. Your child sees the number 18 and that there are 4 uncovered items. They can subtract 18 - 4 and know that 14 pieces are covered. Because the same color is used, the covered items must be equal, so each must be 7, which is half of 14.

Here is where the best, and most important, 'step' starts. What other ways are there of discovering that each covered group has 7 items? Your child could count by 2's and see that there are seven 2's as they skip count from 4 to 18. They could break 18 into halves of 9 each and break the 4 into halves of 2 each – then each group of 9 consists of a covered group plus 2 more, so the covered groups are 7 each.

Keep exploring and thinking of ways to use all the interesting math relationships!

MORE MENTAL MATH – MULTIPLICATION

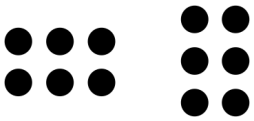
Prerequisite: Comfort with single-digit add/subtract, number bonds, skip counting, doubling

Introduction to Multiplying

Your child already knows how to multiply by 2 by doubling the number. This is an exciting time when your child learns a lot more about multiplication. By the end of this page, your child will be comfortable multiplying the numbers up to 5!

3 x 4 = 4 x 3

Your child is so familiar with addition that it's no surprise that 2 + 3 is the same as 3 + 2. Although not as obvious, the same is true for multiplication.



This illustration shows that two rows of three is the same as three rows of two – you're just changing your point of view!

It doesn't matter which order you multiply two numbers – you get the same answer either way!

It's great that this cool observation means that your child needs to master only about half as many multiplication facts – once your child knows 3 x 4, they also know 4 x 3.

Skip Counting is Multiplication

All that skip counting practice your child did paid off in getting much better at addition and subtraction. It also will be a big help in getting started with multiplication. Skip counting by 5's makes them particularly quick to learn.

Although skip counting is not the fastest way to find a result, it is reliable. To find 7 x 3 either skip count by 3's seven times or skip count by 7's three times.

Your child will eventually memorize these facts, but skip counting is a handy for now.

Multiplying by 3 and 4

For someone good at adding, multiplying by 3 and 4 can be quick and easy.

Multiplying a number by 3 is adding the number to double the number. So, 3 x 6 is 6 more than double 6, which is 6 + 12 = 18.

Four times a number is doubling a number twice. So, 4 x 7 is 2 x (2 x 7), which is 14 + 14 = 28.

ADDING AND SUBTRACTING

Prerequisite: Comfort adding and subtracting single-digit numbers

Sum Square

Puzzle

The setup: Start with a 3 by 3 square grid that has target sums given for each row and column. Some of the numbers from 1 to 9 are already placed in the grid.

The challenge: Place the remaining numbers in the grid to make the row and column sums be the target values.

How to create: To make one of these puzzles, start by placing pieces of paper with the numbers from 1 to 9 on a 3 x 3 grid.

6			14
		4	14
	1		17
15	12	18	

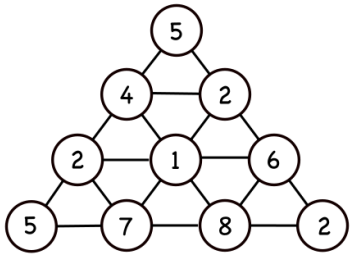
For each row and column, write the sum to the right or below. Then, remove some of the numbers from the grid. Lastly, hand the pieces of paper with the numbers you removed to your child and ask "Where were these?"

Variations

One variation that keeps the sums smaller is to use the numbers from 0 to 8 instead. A harder variation is to do the same thing with the numbers 1 to 12 in a 3 by 4 grid.

Addition Pyramid

Puzzle



Target = 17

The setup: A pyramid of 10 numbers placed in 4 rows is given with a target number.

The challenge: Find a path through the pyramid using one number from each row so that the sum of the numbers is the target number. The numbers on the path must connect to each other.

Example: The answer to this puzzle is 5 –> 4 –> 1 –> 7.

How to create: Make one of these puzzles by filling in the numbers that you want to form the path, and record the sum of those numbers. Then fill in the remaining decoy numbers in the pyramid.

ADDING AND SUBTRACTING

Prerequisite: Comfort adding and subtracting single-digit numbers

Sum Difference

How to play: One person says two numbers, one a sum and the other a difference, and the other person is challenged to find the original two numbers that have that sum and difference. For example, if one person says the sum is 12 and the difference is 6, the other person says the original numbers are 3 and 9.

Reverse roles: Because of how easy it is to create these questions, this is a good activity to let your child be the questioner. Not all combinations of numbers for the sum and difference will produce reasonable answers. However, if you start with two numbers and then say their sum and difference, that will guarantee that there is an answer.

Poison Numbers

The setup: Remove the picture cards from a deck (you can use the Queens as 0's if you like). Before starting, agree on a set of "poison" numbers for the round. The poison numbers can be any set of numbers you want your child to practice with or become more familiar with. Some examples are:

- even numbers (2, 4, 6, 8, 10, 12)
- odd numbers (1, 3, 5, 7, 9, 11)
- square numbers (1, 4, 9, 16, 25)
- prime numbers (2, 3, 5, 7, 11, 13, 17, 19)
- multiples of a number, such as multiples of 3

How to play: Deal each player three cards. The first player discards a number that is not a poison number and replaces it from the draw pile. The next player discards a number so the sum of the first two numbers is not a poison number and replaces the discard from the draw pile. The next player plays so the sum of the three cards is not a poison number, and so on.

How to win: The first player unable to discard a legal card loses and drops out.

Variation

This game works equally well with more than two players.

MAKE IT COUNT

Prerequisite: Comfort adding and subtracting single-digit numbers

Introduction

Let your child play with and think about these investigations. Don't be in a hurry. The value is in the playing around and discovering beautiful patterns. There is some deeper mathematics in these activities, but those ideas can wait until your child is older.

Flower Petals

Investigation

The question: In a peculiar garden there are two kinds of flowers. One has 4 petals and the other kind has 7 petals. A child was asked to pick some flowers so that the total number of petals was 13. Could ibt be done? How about 15 petals? For which numbers of petals is it possible? For numbers that are possible, can it be done in more than one way? For example, 32 petals is four 7's and one 4, and it is also eight 4's.

Variations: By varying the numbers, there are lots of examples to play with. For some pairs of numbers there comes a point where all numbers of petals are possible, and for other pairs of numbers there is no such point. For 4 and 7, every number from 18 on is possible. For 3 and 6, there is no point after which all numbers occur.

Climbing Steps – How Many Ways?

Investigation

The question: Suppose your child likes to take steps two at a time sometimes, and one at a time other times. If your child goes up some steps, how many ways can this be done?

For example, for 0 steps there is one way - you stand there. For 1 step there is one way. For two steps, you can either take one double step or two single steps, so there are two ways.

Carefully think through many examples and then make a table of the results. When there is lots of information, making a table often helps. The start of the table looks like this:

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	5	8	13	21	34	55	89

After looking at these numbers, your child may notice that each pair of numbers adds up to the next number. Why does this happen? These numbers are called Fibonacci Numbers.

MAKE IT COUNT

Prerequisite: Comfort adding and subtracting single-digit numbers

Balance Scale

Investigation

A balance scale is a device for telling when two things have the same weight. The scale is usually supplied with a set of weights that are used to weigh objects. Here are some interesting investigations you can do if you restrict the weights you are allowed to use.

Question 1: If you only have weights that are 4 units and 7 units, then the things you can weigh exactly are the same as you found in the flower petal investigation.

Question 2: Which things can you weigh exactly if the weights are on either side of the scale?

Question 3: How do the answers of these two questions change if instead of using weights of size 4 and 7, you use 3 and 8? Or perhaps you use 4 and 6 or 8 and 12?

Question 4: What happens if you have one weight each for each of the weights in a doubling progression of 1, 2, 4, 8, and 16? How many ways can you weigh something that weighs 13? Does it change things if you allow weights on both sides? What is the largest weight you can measure? This situation is related to the binary number system.

Question 5: What happens if you use single weights in the tripling progression 1, 3, 9, and 27? Which things can you weigh if you allow those weights on both sides?

Question 6: What happens if the weights are the Fibonacci Numbers? Is there more than one way to weigh some weights? Find a restriction on the Fibonacci weights so that there is only one way to get each weight.