

## Stage 3 – I Can Count to 10!

**Prerequisite:** Ability to count to 10 and have a sense of those quantities. Can add numbers from 0 to 5 using manipulatives, especially fingers. Can add or subtract 1 and 2. Also has knowledge of, and can reason with, properties of objects such as color, shape, and texture.

### Where You've Been . . . . .

Your child counts up and down between 0 and 10 and understands what those quantities mean. Early addition and subtraction skills are developing. An important foundation for those skills is confidence with adding and subtracting 1 and 2 with small numbers. Your child understands small quantities, and reasons with those quantities to do addition and subtraction for small numbers.

In addition to that wonderful stuff, your child reasons so much better now! They understand that objects and numbers have properties, and they can reason and do beginning problem solving. Your child is now a full member in family math games and puzzles and exploring the mathematical world around them

### New Ideas in this Stage . . . . .

- **Counting On** – Count upward starting at any number, rather than always starting at 1. This is useful for addition and for finding differences.
- **Counting Down** – This refers to counting downward starting at any number. It is useful for subtracting, as well as for developing a sense for the relationships between numbers.
- **Number Bonds** – These are all pairs of numbers that add up to a specific number.
- **Ten Frames** – Represent a number from 0 to 10 as that number of dots inside a 2 by 5 rectangular grid. For numbers greater than 4, the upper group of 5 squares is always filled.
- **Expanded Form** – This refers to writing a multi-digit number broken down into the contribution of each of its digits. For example:  $25 = 20 + 5$  and  $317 = 300 + 10 + 7$ .
- **Fact Families** – This refers to a group of closely related math facts. For example,  $2 + 5 = 7$  is in the same family as  $7 - 2 = 5$  and  $7 - 5 = 2$ .
- **Adding twins and near twins** – An adding twin is adding a number to itself, such as  $4 + 4$ . A near twin is one away from a twin, such as  $4 + 5$ .
- **Doubling, multiplying by two, and halving, two equal parts, dividing in two** – Children usually enjoy adding twins. With that comes the idea of doubling and multiplying by 2. Also associated with that is halving, splitting something into two equal parts, and dividing by two.
- **Even and Odd numbers** – Even numbers can be split into two equal parts. Odd numbers have one left over when split into two equal parts. Even numbers are the results of adding twins.
- **Skip counting by 2's** – Count up or down by 2's – such as, 0, 2, 4, 6 or 13, 11, 9, 7.

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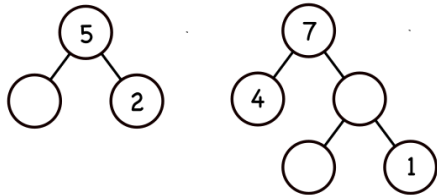
# FACT FAMILIES

**Prerequisite:** Some comfort adding and subtracting small single-digit numbers

## Shape Sums .....



Puzzle

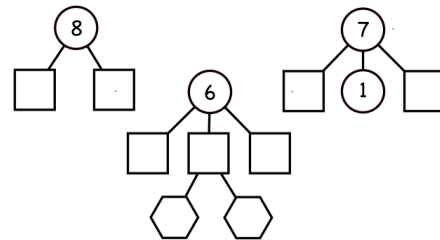


**The challenge:** Fill in the missing circles so that every circle is the sum of all the circles directly below and connected to it.

The easiest puzzles have most of the circles filled in. For older children, there are variations that involve larger numbers and cleverer solutions.

**Repeated numbers:** One option is to use non-circular shapes for repeated numbers. While the value in a circle may duplicate the value in some other circle or shape, the value in a non-circular shape must match the value in all other places with the same shape.

For example, all squares have the same value in a given puzzle. Use matching shapes to practice adding twins, near twins, and halving – in the first example, the solver is asked to find a number that is half of 8.



**How to create:** Make these puzzles by starting with a diagram that is completely filled in and then removing some numbers. If the puzzle has some repeated numbers, use a square, triangle, or other shape instead of a circle for that repeated number.

## Mystery Change .....



Activity

**How to play:** Have your child count some small number of objects. While they look away, change the number of objects. When they look back, ask what change you made. They can test their theory by reenacting what they think happened.

### Variations

*Once this is easy, you can have them be more creative with their answers. For example, if 4 became 6, the answer might be that you doubled the 4 and then took 2 away.*

# ADDING AND SUBTRACTING 10








*Prerequisite: Comfort counting to 10, some comfort counting to 20*

## Introducing 10's . . . . .

Welcome to the world beyond 10 fingers! There are wonderful things to discover here. The next group of numbers from 10 to 20 is 10 more than the numbers your child already knows. Before this can become easy, your child needs to conquer the crazy names we use for eleven, twelve, and thirteen.

The games on the next page are designed to emphasize the role that 10 plays in connecting pairs of numbers such as 6 and 16. These games also emphasize the idea that 16 should be thought of as 10 plus 6. This view of decomposing numbers using place value will be much more important as your child counts to 100 in the next Stage.

## Making Number Cards 0 to 20 . . . . . Activity

0	1	. . .	9	10	11	. . .	19	20
0+	0+	. . .	0+	10+	10+	. . .	10+	20+
0	1	. . .	9	0	1	. . .	9	0
		. . .				. . .		

If you don't have them already, create some extended decks of counting cards from 0 to 20. One deck will be normal numbers, one deck will have the numbers in expanded form from 0 to 20 as 0 + (0 to 9), 10 + (0 to 9), and 20 + 0, and one deck will use ten frames.

# ADDING AND SUBTRACTING 10

**Prerequisite:** *Comfort counting to 10, some comfort counting to 20*

## Bingo With 10 .....



**The setup:** Place a random collection of 16 Number Cards from 0 to 20 with expanded form on a 4 by 4 bingo board for each child.

**How to play:** Generate random numbers to be called out one at a time. Mix up a collection of counting cards from 0 to 20. Select one card at a time from this pile until the first child gets four in a row and yells Bingo!

### Variation

*One important variation of this game is to do a “Tens-Reversed” version using cards with numerals. When a card is chosen, if it is 1 to 10, then 10 is added to find the matching value, and if it is 11 to 20, then 10 is subtracted for the matching value.*

## Memory Challenge – 10's .....



**The setup:** This version of the Memory Challenge game uses a Number Card deck from 0 to 20 with the rule that two numbers match if they are 10 apart. If you also have cards from 0 to 20 that use expanded form or ten frames, you should use those too. Deal a 3 by 4 grid of cards out on the table, all face-down.

**How to play:** Players take turns flipping two cards face-up. If the two cards are ten apart, the player gets to keep the cards, replaces the two cards from the draw pile, and continues their turn. If the cards do not match, the player flips the cards back over and ends their turn.

**How to win:** The game ends when the last pair of cards is taken. The player with the most cards wins.

# SHAPES

*Prerequisite: Comfort counting to 10; some comfort counting to 20*

## Geometric Art .....



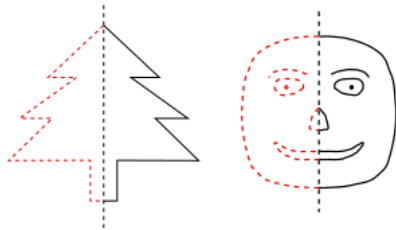
Activity

Similarity and symmetry are two geometric concepts your child can play with.

**Similar shapes:** Two shapes are similar if they have the same shape, where one may be smaller or larger than the other. Circles have the same shape, so all circles are similar. This sequence of three stars are similar.



Challenge your child to pick a picture and draw it two or three times as big, or twice as small.



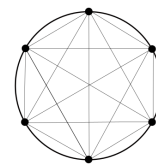
**Mirror symmetry:** Another geometric concept to play with is mirror symmetry. Show this using a mirror with a flat side – put it down along its edge on a drawing or photo and see what the mirror image looks like. Once your child has the idea, give your child half a picture and challenge them to draw the mirror image.

## Sim Triangle .....

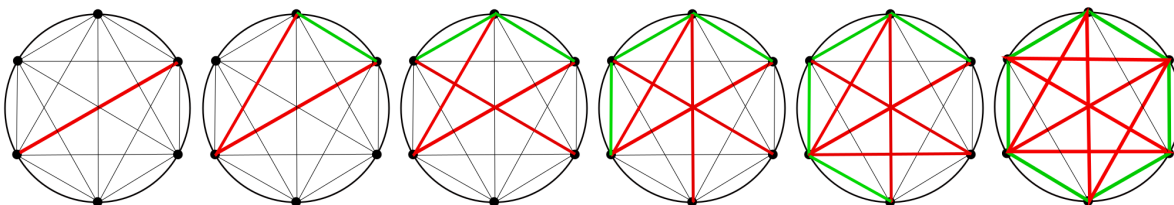


Game

**How to play:** The two players have different colored markers. Place six (use more for a harder game) dots evenly around a circle. Players take turns drawing lines between the dots using their color. The loser is the first player forced to create a triangle all of whose sides have the player's color and whose corners are on the circle.



**Example game:** Here is a game with 'red' moving first. This is a sequence of board positions after each of red's moves. In the last position, no matter where 'green' moves next they will form a triangle and lose.



# NUMBER SHAPES

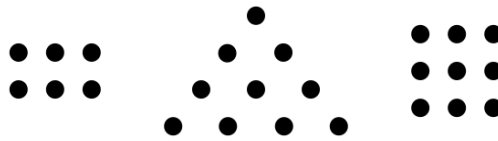
*Prerequisite: Comfort counting to 10, some comfort counting to 20*

## Number Shapes



Investigation

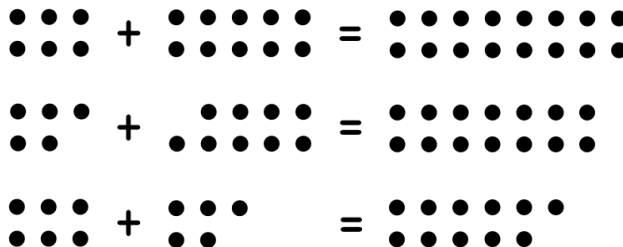
Using something small, such as pieces of food, challenge your child to make shapes with a given number of pieces. These shapes can be rectangles, triangles, squares, or anything fun.



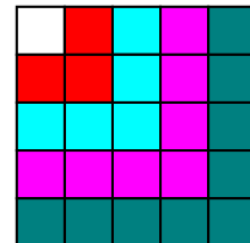
**Even and Odd:** Investigate even and odd numbers using number shapes. For a given number, ask your child to put the pieces into two equal rows. You would do this if you were splitting food fairly between two people. For which numbers does it work out evenly?



**Adding Evens and Odds:** Representing even and odd numbers this way makes it easy to see why: Even + Even = Even, Odd + Odd = Even, and Even + Odd = Odd.



**Adding up odd numbers:** Once your child knows what an odd number is, investigate adding up the first few odd numbers as shown in this diagram. Amazingly, the sum of the first odd numbers is always a square number.



**Making rectangles:** Your child may notice that for some numbers, such as 12, there are different shapes of rectangles that can be made, and that for other numbers, such as 7, only flat rectangles can be made. Numbers such as 5 and 7 are called prime numbers because there is no way to break them into normal rectangles.

# Evens AND ODDS

**Prerequisite:** Some comfort counting to 20; add and subtract 1 and 2 easily

## Multiplying by 2 . . . . .

Very gradually, introduce multiplication to your child by a small change in language – start referring to doubling a number as taking “two of it” or as “multiplying it by 2.”

## Nim – Doubling the Limit . . . . .



Game

**The setup:** Set a starting total, say 20. Let your child choose whether to go first or second.

**How to play:** During the first turn, a player chooses to subtract 1 or 2 from the current total. After the first turn, a player may subtract any number from 1 up to twice the number used on the last turn.

**How to win:** The first person to reach 0 wins (an alternative rule is they lose).

### Variations

*This can also be played, with essentially the same rules, by starting at 0 and adding your way up to the target. Once children learn to play this without writing anything down, it is a fun travel game.*

## Halves and Half Nots . . . . .



Game

**The setup:** Two players agree on a target number, say 20, set their running total to 0, and choose who will go first.

**How to play:** A turn starts by generating a number using one die or the sum of two dice. If the number is odd, the player must double it. If the number is even, the player may take half of it as many times as is desired as long as the numbers being halved are even. The player then adds that final result to the running total as long as that does not put the total over the target - if the final result cannot be used, the turn is skipped.

**How to win:** The player who brings the total exactly to the target number wins.

### Variations

*You can allow the player not to change the initial number. You can allow halving at most once. Finally, you can practice subtracting by starting at the target number and subtracting your way down to 0.*

# EVENS AND ODDS

**Prerequisite:** Some comfort counting to 20; add and subtract 1 and 2 easily

## Counting Evens and Odds . . . . .



**The setup:** Use a small collection of playing cards involving some small quantities. Start with combinations of three cards and work your way up to more cards.

Suppose the numbers are 1, 2, and 3. The question is: If you randomly pick two cards and add them, are you more likely to get an even or odd number? Count how many ways there are of getting an odd number versus an even number. For example, in the case of using 1, 2, and 3, there is one way to get an even number ( $1 + 2$ ) and two ways to get an odd number ( $1 + 3$ ,  $2 + 3$ ). So the odd number sums are more likely (twice as likely).

**Make it a contest:** Let one player be Even and the other player be Odd. See who has the most successes after a dozen trial runs.

## Skip Counting by 2's . . . . .



In addition to being good addition practice, skip counting is a faster way to count a collection of things, such as toes, than counting them one at a time. Skip counting is also very helpful when learning multiplication later on.

**Alternate counting turns:** Count by 2's while pushing your child on the swings (or anything else that involves counting). Start by alternating with your child – you say 1, your child says 2, you say 3, your child says 4, and so on. After establishing the pattern, one of you can say their part without the other person saying anything.

While traveling, find something fun to skip count like yellow cars.

### Variations

Skip count up **or down** by 2's ending by 20.

Initially, do this starting at 0 or 20, but eventually, start at any number.

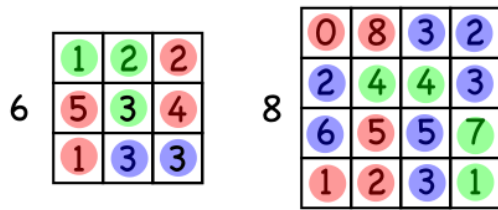
# NUMBER BONDS

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Sum Groups .....



Use a rectangular grid of numbers with a target sum of your choosing between 5 and 12.



**The challenge:** Find groups of two or three numbers that add up to the target and share sides. When complete, the entire puzzle will be made up of identified groups. Use tokens, such as different types of food items, to identify each group within the puzzle.

**Puzzle creation:** Create these puzzles by starting with an empty grid and working your way around the grid using pairs and triples that add up to the target sum. It's more fun if the puzzle has just one solution, but it's okay if it doesn't.

## Go Fish With Sums .....



**The setup:** Using a target sum that your child is comfortable with, remove cards at or above that target from the Number Card deck. If there are more than two players and you have restricted the numbers significantly, you may need to use several decks.

**How to play:** Play starts by dealing 5 cards to each player. Put the remaining cards in a common draw pile. Players take turns "fishing" for cards whose number will sum to the target sum with cards they already have.

For example, a player might ask a player, "Do you have any 4's?" If that player has some 4's, they are handed over and the original player gets another turn. However, if that player does not have any 4's, then the player says "Go Fish!" and a card is drawn from the draw pile. If the drawn card matches with a card they have, the player may have another turn; otherwise, the turn is over and play continues to the left. When a player has a pair of cards that sum to the total, the player puts that pair on the table in front of them.

**How to win:** The game is over when all cards are in pairs. The player with the most pairs wins.

### Variations

*To create some variety, allow players to use more than two cards to create a group of cards that sum to the target sum. Another possibility is to say that two cards match when their difference is a specified target difference.*

# NUMBER BONDS

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Which Number am I? . . . . .



Depending on whether there are one or two children playing, there are two ways to play.

**Two children:** Each child draws a card and places it on their forehead facing out without seeing it. You announce the sum of the cards and the children are challenged to figure out their own card by looking at the other child's card.

**Adult with one child:** Create several pairs of cards in advance where each pair of cards has the same total known to all. The unused cards are moved to the side. Pick up one random pair of cards, place the cards on your foreheads, and figure out each card by looking at the other person's card.

## Chopsticks Hand Game . . . . .



**The setup:** All players start with one finger raised on each hand.

**How to play:** During a turn, a player has the choice of either "attacking" or "splitting."

**Attacking:** To attack, a player takes a live hand and attacks a live hand of an opponent. The result is that the opponent's hand has the sum of the two hands and the attacking player's hand is unchanged. If a hand ends up with exactly five fingers, it is dead. If the hand has over five fingers, its count is either reduced by five (in one set of rules) or is dead (an alternate set of rules).

**Splitting:** To split, a player bangs their hands together and redistributes the fingers between the two hands. A split may not reverse the two finger counts.

**How to win:** A player wins when both hands of everyone else are dead. In one variation, the first player to have two dead hands wins.

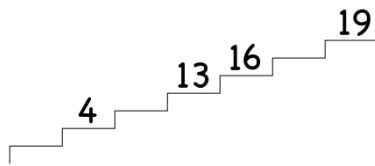
# ORDER

**Prerequisite:** Can count up and down from 0 to 20

## Going Up . . . . .



**The setup:** Each player draws a staircase with anywhere from 4 to 10 steps (more steps for older players).



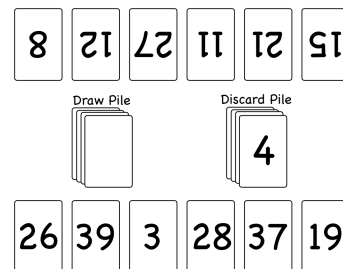
**How to play:** Create a two-digit number using cards numbered from 0 to 9. Determine the tens digit by selecting from cards 0 and 1, and then determine the ones digit by selecting from all the cards. If possible, the player must place this number on their steps.

**How to win:** The first player to fill their steps in increasing order from bottom to top wins. Make it easier by allowing neighboring steps to have the same value.

## Creating Order . . . . .



**The setup:** Start with a deck of Number Cards that goes from 0 to 40 (higher with more players). Deal ten cards to each player (as few as four for younger players). The dealt cards are placed in front of each player in the order dealt. The remaining cards are placed face down in a draw pile. Place the top card face up as the first card in a discard pile.



**How to win:** The goal of the game is to get the cards in increasing order from left to right.

**How to play:** During a turn, a player may select either the top card of the discard pile or the draw pile - this card must be used to replace one of the cards in front of the player, and the replaced card is put face up on the top of the discard pile.

### Scoring variations

*Play that the first player to get their cards in order wins. Or, use a point system for each round of play. In this system, the winner receives 15 points. The other players receive one point for every card they have in order starting with their lowest card.*

# SMALL ADDING AND SUBTRACTING

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Closest to 10 .....



**The setup:** Use a deck of cards from 1 to 10. Choose a target sum, say 10.

**How to play:** Each round starts by dealing five cards face down on the table. One player randomly chooses three of those cards and turns them over. Two cards are chosen from those three to add up as close to the target as possible. The unused three cards are given, face up, to the other player to choose two cards to get close to the target.

**Scoring:** The player whose result is closest to the target from either side wins a point. Have a number line handy to use for discussions about which result is closer to the target sum.

### Variation

*If you use subtraction instead of addition, use a lower target number, say 4.*

## Zoo Rescue .....



**The setup:** Use two dice or two sets of number cards going from 1 to 6. Each player has 6 tokens – animal tokens are perfect for this game if you have them. Each player also has a piece of paper with boxes numbered from 0 to 5. Each player decides where to put their 6 tokens – it is okay to put more than one token in a box.

**How to play:** During a player's turn, two numbers are created by rolling the dice or picking two cards, and the difference of those numbers is used. A player can free one of their tokens if they have one in that box.

**How to win:** The first player to rescue all their tokens wins.

**Strategy:** An important part of this game is choosing good boxes to place tokens in to start with. With practice, your child will realize that some boxes are more likely to occur than others – you can discuss ideas about why this is.

### Variation

*Use cards numbered from 1 to 10 and have 10 boxes numbered from 0 to 9.*

# NUMBER BONDS

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Hot Potato with Sums . . . . .



**The setup:** Use a regular deck of playing cards, have a target number of 5, and remove all cards above the target, except for one card – the 'hot potato,' which can be anything else, such as a Joker or face card. Deal the entire remaining deck to all players – it's okay if some players get one more card than others.

**How to play:** All players start by placing on the table from their hands 5's and any pairs that add up to 5. On your turn, you can pick a random card from any other player's hand (a simpler rule is that it is always the player on your right). If this new card gives you a pair of cards that add up to the target sum, place the pair on the table; otherwise, keep the card in your hand. Play then moves to the next player on your right.

**How to win:** When play ends, the player holding the hot potato loses. Alternatively, you can play that the first player to run out of cards wins.

### Variations

*As your child's adding skills improve, use larger target numbers up to 10. Varying the target number will provide practice with the various number bonds.*

## Common Sums . . . . .



Investigation

**Collect the data:** Make a sheet of paper with 12 rows. In each row, put 8 squares. The leftmost column of squares has the numbers from 1 to 12 written in order in the squares. Put 1 token on each of the 12 numbers. Start rolling a pair of dice. After each roll, move the token for the sum of the dice one square to the right. The goal for each token is to be the first to get all the way to the right across the page.

Let your child come up with some questions to investigate. Some natural questions are:

- Which token(s) will win and why?
- Which tokens do well and which ones do poorly?
- Which token is the worst?
- How will the winners change if the row lengths are made shorter or longer?

Have your child explain their ideas about the answers to these questions, and then investigate their ideas by running experiments.

Add a competitive element to this by guessing which token will win before the round starts.

# SUDOKU VARIATIONS

**Prerequisite:** Some comfort counting to 20; can easily add and subtract 1 and 2

## Introduction . . . . .

The puzzles on this page all have the basic Sudoku rule that a square grid is filled with numbers so that each number appears exactly once in each row and column.

Make these puzzles by starting with a completely filled puzzle, removing many or all the numbers, and creating subregions and the appropriate extra information for that type of puzzle.

To save space, the examples here are all 4 by 4. You can make larger puzzles yourself, or you can find larger versions of these on the internet.

## Even-Odd and Kropki Sudoku . . . . .

Puzzle

These puzzles have the additional restriction that the numbers must occur exactly once in each marked 2 by 2 subregion.

1			4
		1	
		2	1

1			

**Even-Odd Sudoku:** Squares with even numbers are greyed in.

**Kropki Sudoku:** Hollow dots between cells mean the numbers are one apart; filled dots mean one number is half the other.

## Sumdoku, Diffdoku, SumDiffDoku . . . . .

Puzzle

In addition to the usual Sudoku rules, these puzzles are broken into subregions with target numbers. Unlike standard Sudoku, it is allowed for a number to be repeated in a subregion. If a subregion has just one square in it, the target number will be the value of that square.

**Sumdoku:** The sum of the numbers in a subregion is the target number.

**Diffdoku:** Subregions have one or two squares. For two squares, the difference of the numbers is the target number.

3+		3	7+
6+	4+		
		6+	4+
7+			

3-	1-	3	2-
		3-	
1-	1		2-
	2-		

**SumDiffdoku:** The subregions are marked with a "+" or a "-" to indicate sum or difference.

### Variations

Use different groups of numbers instead of the usual 1 to 4 for a 4 by 4. For example, use 1, 3, 5, and 7. If you do this, list the numbers to use above the puzzle.

# SUM COMBINATIONS

**Prerequisite:** Some comfort adding and subtracting small single-digit numbers

## Shut the Box . . . . .



**The setup:** Each player writes the numbers from 1 to 9 in a row on a piece of paper.

**How to play:** To start a turn, a player finds the sum of the roll of two dice. Using only numbers that have yet to be crossed out, the player crosses out a group of one or more numbers that add up to that sum. If this can't be done, nothing changes. A player may decide in advance to use just one die.

**How to win:** The first player to get all numbers crossed out wins.

### Variations

*One way to vary this game is to use a larger range of numbers, such as going to 10 or even 12. Another way is to give each player a single turn - the turn continues with new rolls until the first time the player is stuck. At the end, the player's score is the sum of the numbers not crossed out. The player with the lowest score wins.*

## Hit the Target . . . . .



**The setup:** From a deck of playing cards, remove the face cards and any numbers that are larger than what the children are comfortable with. Shuffle the cards and turn over five cards to use and a sixth card that is the target. Leave the remaining cards as a draw pile to fill in cards as they get removed.

**How to play:** During a turn, if a player can use the sum or difference of two of the five cards to equal the sixth, the player gets all three cards and they are replaced from the draw pile. If the player fails, then a new sixth card is turned over and the turn moves to the next player.

**How to win:** The player with the most cards at the end of the game wins.

### Variations

*There are several variations you can use. You can allow a player to use three cards, instead of just two, to add up to the sixth. You can even allow any number of cards to be used to add up to the sixth. Another variation is to allow a mixture of addition and subtraction with any number of cards.*

# LET ME COUNT THE WAYS

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## How Many Ways? . . . . .



Investigation

Counting the number of ways of doing something involving choices can lead to some interesting investigations. Here are a few possibilities – have fun thinking of many more with your child.

**Investigation 1:** Drawing with only red and blue, how many ways can you draw a monster with a hat, eyes, and cape? How does this change if you only colored the hat and the cape? How would it change if you used three colors, or if you could only use each color once?

**Investigation 2:** You have a row of 5 identical candies. How many ways can you color them so that there are 2 red ones and 3 blue ones?

**Investigation 3:** Find all the ways to get a sum using a small set of numbers. Do this with and without considering order. For example, if you use 1 and 2, there are  $1+1+1+1 = 2+1+1 = 2+2$  ways of getting 4 without considering order, and  $1+1+1+1 = 2+1+1 = 1+2+1 = 1+1+2 = 2+2$  ways of getting 4 considering order.

## How Can I Make? . . . . .



Activity

One person gives the other person a target number and challenges them to come up with different ways of adding and/or subtracting to get that number. If the target is 3, it might start with  $1 + 2$ , but then become  $1 + 5 - 3$  and so on.

Add restrictions to make it more challenging. For example, you might not allow any numbers less than 4, or you might allow only numbers less than 4.

You can turn this into an investigation by asking questions like: How many ways are there of getting 5 if I can only add 1's and 2's.

As your child learns more mathematics, additional skills can be added, such as doubling and halving. For example, 5 is 2 doubled plus 1 or two 10's minus three 5's.

# PUTTING THE PIECES TOGETHER

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Number Scramble . . . . .



**The setup:** Roll one die twice to create a two-digit target number – the first roll will be the tens digit and the second roll the ones digit. Roll five dice to create numbers to work with.

**The challenge:** Use addition, subtraction, and forming two-digit numbers to get as close as possible to the target number – the score is how close they come.

**How to win:** You can either have a winner for each round, or you can add up the scores over several rounds and have an overall winner with the lowest score. Of course, you can play it for the challenge and not keep score.

**Example 1:** Suppose the roll is: 4, 4, 3, 1, 3 and 22. The first player gets  $4 + 4 + 3 + 1 + 3 = 15$ ; another player gets  $14 + 4 + 3 + 3 = 24$ ; and another player gets  $34 - 14 + 3 = 23$ .

**Example 2:** Suppose the roll is: 1, 2, 5, 6, 4 and 63. A player gets close with  $65 - 4 + 2 + 1 = 64$ . Another one gets it exactly with  $56 + 4 + 2 + 1 = 63$ . A third says  $52 + 6 + 4 + 1 = 63$  also works. Have a chat about why some problems have lots of solutions.

### Variation

*Include multiplication once your child knows how to double or triple numbers.*

## Any Which Way . . . . .



The players agree on a target. A useful target is 10, but other numbers are fine.

Each player is dealt five cards from a deck without face cards. Players then find as many ways as they can to get the target, or get as close to the target as possible, using any combination of addition and subtraction with any of their cards.

Do this as a group activity where the whole group is given five cards to use to cooperatively come up with ways to achieve the total.

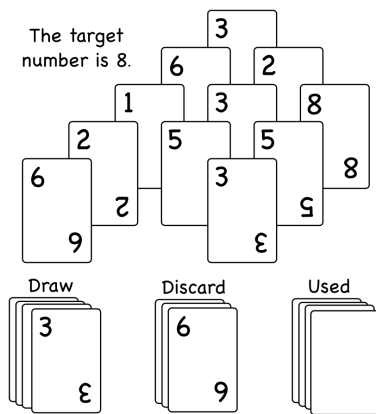
# SOLITAIRE PUZZLES

*Prerequisite: Some comfort adding and subtracting small single-digit numbers*

## Introduction . . . . .

While these puzzles are described as solitaire puzzles, they are best done together as a family.

## Pyramid Solitaire . . . . . Puzzle



**The setup:** Remove cards above a target number, say 10. Make a 21-card pyramid with 6 rows. Place the cards in the pyramid face up, with each lower row half-overlapping the row above it. The remaining cards form a draw pile.

**How to play:** Collect, and move aside, uncovered groups of cards that sum to the target. This group can include the top card of the draw pile or the discard pile. If there are no matches, move the top draw pile card to the discard pile.

**The challenge:** Empty the pyramid of all its cards. The game is over when the draw pile is empty.

**Lower targets:** If you use a target less than 10, reduce the pyramid size by a row or two so that you have a big enough draw pile. For example, for a goal of 8 use 15 cards in 5 rows.

**Use subtraction:** Instead of using addition, use subtraction. If you use the cards up to ten, a good target number is 5 – remove pairs of cards whose difference is 5. When you have a '5,' you can either remove it or pair it with a 10.

## Difference Pyramid . . . . . Puzzle

**The challenge:** The challenge is to put the numbers from 1 to 6 in a pyramid with one card in the top row, two cards in the second row and three cards in the third row, where each number is the difference of the two numbers directly below it.

Let your child take time and play with this. Most likely, just by playing around, they will happen upon a solution. However, if your child gets anxious, here are two tips that help. The 6 must be in the bottom row because it cannot be the difference of any pair of numbers. Similarly, the 5 must either be in the bottom row or in the middle row above the 6 and the 1.

**Related challenges:** If your child finds this easy to do, challenge them to find how many ways it can be done. Discuss what it means for two solutions to be different – if one solution is the mirror image of another, should it be considered different? Another challenge is to solve the harder puzzle of putting the numbers from 1 to 10 in a four-level difference pyramid.

# STRATEGY GAMES

**Prerequisite:** Reasoning and problem solving skills

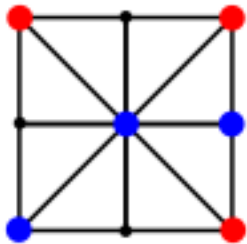
## Introduction . . . . .

Playing strategy games will help your child with many reasoning, planning, and problem solving skills. There are dozens of such games – such as tic-tac-toe, 9 Men's Morris, Fox and Geese, Dara, Connect Four, Mancala, Checkers, Go, Chess – far too many to cover here.

## Tapatan . . . . .



**The setup:** There are many games similar to this one – games such as Rota, 9 Holes, and 3 Men's Morris. In this game, there are 9 points connected in a grid by 3 horizontal and 3 vertical lines. Each player has three identical tokens to play with. The goal of the game is to get your three tokens in a line.



**How to play:** There are two phases of play. During the first 'placement' phase, the players take turns placing their tokens one at a time on vacant points. After placing the tokens, the second 'movement' phase begins. During this phase, players take turns moving their tokens to empty adjacent points. An alternative rule is that pieces can be moved to any empty point, not just adjacent points.

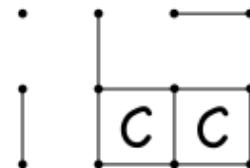
**How to win:** The game is over when one player wins or when the same position occurs three times, in which case it is a tie game.

## Dots and Boxes . . . . .



**The setup:** This satisfying game starts with a rectangular grid of dots – say 3 by 4.

**How to play:** During a turn, a player draws a horizontal or vertical line connecting two unjoined adjacent dots. If the new line completes a 1 by 1 box, the player earns a point and draws another line.



**How to win:** When no more lines can be drawn, the player with the most points wins. An easy way to keep track of points is to put an initial inside each earned box.

### Variation

*Although this is traditionally played with little squares, it can also be played with dots in a pattern to produce triangles or hexagons – it's just harder to draw.*