

## Stage 2 – I Can Count to 5!

**Prerequisite:** Your child can count to 5 comfortably and has a sense of those quantities. Your child also has some knowledge of properties of objects such as color, shape, and texture.

### Where You've Been .....

Wow! Your child is now able to count to 5! These numbers are not just being repeated in order like a parrot – they represent quantities that your child is gaining a feel for. You are ready to assign properties to objects that you describe and talk about. Think how far your child has come in understanding the world!

Your child is becoming more verbal and better able to reason and explain. You can discuss mathematical things and this allows you to start playing games and doing puzzles together.

### New Ideas in this Stage .....

Your child will be learning many things in the coming months beyond counting to 10 and above. Here is a quick list of topics that will be covered in this Stage.

- Counting forward and backward to 10. Don't ignore backward counting – it is important for understanding number relationships as well as being an aid for subtraction.
- Include 0 in your counting sometimes. Including 0 now makes it a familiar number and keeps it from feeling exotic.
- Reasoning with properties and numbers. Understanding properties and reasoning with them is a key part of developing mathematical ability.
- Deepening understanding of shapes, especially circles, triangles, and squares. Keep using descriptive shape names as your child interacts with them.
- Comparing and ordering numbers. How quantities compare and interact with each other is central to understanding them.
- One more, one less, two more, two less. These concepts are relatively easy to pick up, and they will form the basis for addition and subtraction.
- Learning to use manipulatives, especially fingers, to understand adding and subtracting. Adding and subtracting 0.
- Subtracting a number from itself.

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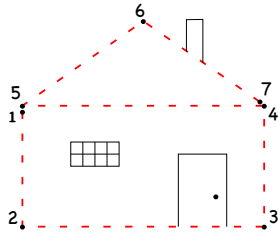
# COUNTING UP TO 10

Prerequisite: Counting up to 5; Number Cards

## Connect the Dots . . . . .



Activity



**How to create:** Complete fun drawings by connecting numbered dots. Make these by taking a simple drawing, removing some straight lines, and replacing them with numbered dots. Connecting the dots in order recreates the original drawing.

**Backwards:** Challenge your child to connect the dots in backwards order.

You can also find these drawings to download from the internet.

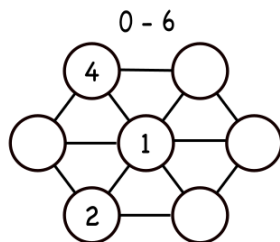
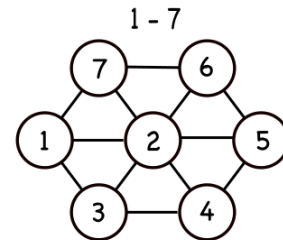
## Island Hopping – Counting . . . . .



Puzzle

These puzzles use numbered islands (circles) connected by bridges (lines).

**The challenge:** Find a path that connects the islands in order. The easiest versions have numbers that go from 1 to the number of islands.



**Adjust the challenge 1:** Leave out some numbers so your child figures out what is missing and where it belongs.

**Adjust the challenge 2:** Instead of starting at 1, design the puzzles to start at 0 or other numbers. (Note the range at the top of the puzzle).

### Another way to play

Make this into a physical puzzle by placing pieces of paper with consecutive numbers on the floor in some twisty path. Your child can solve this puzzle by walking along the path from the smallest to the largest number.

Add challenge: Replace some of the numbered pieces with blank ones. Also, have your child to start at the largest number and go downward.

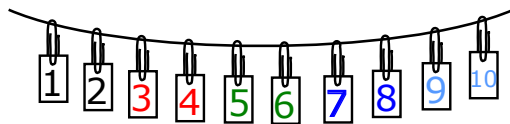
# COUNTING UP TO 10

**Prerequisite:** Counting up to 5; Number Cards

## String Number Line .....



Activity



In addition to having a paper number line on a wall, create a number line using a piece of string. Tie or clip a section of string between two objects. Use paper clips to attach sliding Number Cards from 0 to 10 along the string in order.

### Ideas to explore

*In addition to exploring these ideas, you and your child may discover many more.*

- Switch two numbers and have your child find the mistake.
- Leave out a number and have your child find which one is missing.
- Practice adding. For  $4 + 2$ , slide over the first 4 numbers and then slide the next 2.
- Practice subtracting. For  $6 - 2$ , slide 6 cards left, then slide 2 of those to the right.

## GO FISH! .....

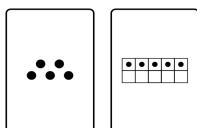
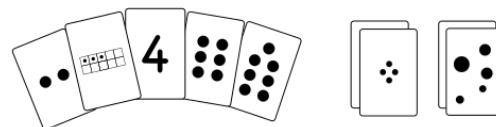


Game

**The setup:** Remove cards above the highest number your child is comfortable with. For more than two players, if you have restricted the number range you may need several decks.

**How to play:** Deal 5 cards to each player and put the other cards face down in a shared draw pile.

During a turn, a player “fishes” by asking any player if they have a card matching one of their cards. For example: ‘Jonah, please give me a 4’. If they have it, they hand over the card. If they don’t, they say “Go Fish!” and the player must pick a card from the draw pile.



**Making a book:** When a player has a pair of matching cards, those cards are placed in a “book” in front of them.

**How to win:** The game is over after all the cards have been put into books. The player with the most books wins.

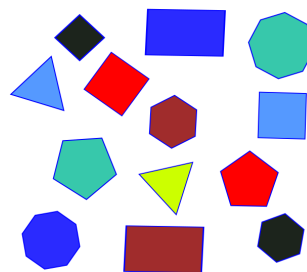
# SHAPES

*Prerequisite: Counting up to 5; Beginning sense of shapes*

## Shapes on the Floor . . . . .



**The setup:** Cut large shapes from big pieces of paper (use color paper if you have it) and place the shapes on the floor. At first, use basic shapes such as triangles, rectangles, squares, pentagons, hexagons, and octagons. Find patterns for these online or in the EFM Printables file. To cause more running around, include more than one of each shape.



**The challenge:** Give your child some shape information and ask them to run to that shape, or shapes. For a very young child, show them a drawing and ask them to find and name that shape on the floor. For older children, name the shape and challenge them to find it.

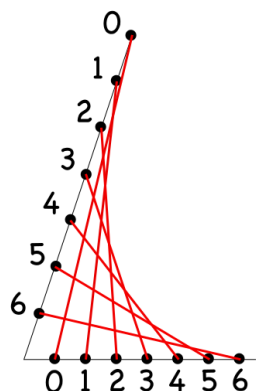
Add variety to this by asking for shapes all of whose sides are the same length, or all of whose angles are the same (or all different), or whose opposite sides (or angles) are the same size.

**Extending the challenge:** With experience, include less basic shapes – add specific triangles (right, obtuse, acute), a kite, a parallelogram (diamond), a star, and some unusual shapes.

As your child gets experienced, playfully mix in some impossible requests such as a triangle with two right angles or a quadrilateral with exactly three right angles.

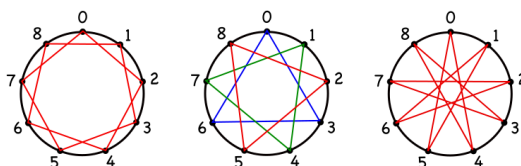
**Another way to play:** Reverse roles so your child makes up questions and you find the shapes. Make a "mistake" sometimes and have your child explain to you what you did wrong.

## Connect the Dots – String Art . . . . .



Make abstract drawings by connecting points with the same number along opposite sides of an angle.

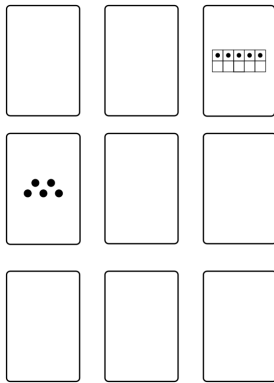
Another way is to put some dots, say 9, evenly spaced on a circle. Play with creating different patterns by connecting the dots in order, or connecting every second dot, or every third dot.



# NUMBERS UP TO 10

**Prerequisite:** Counting up to 10; Number Cards

## Memory Challenge .....



**The setup:** Select two or four sets of Number Cards and remove numbers above your child's comfort level. Place the cards face down in a 3 by 3 grid, with the remaining cards put in a draw pile.

**How to play:** Take turns flipping two cards face-up. If the cards match, the player gets to keep the cards, replaces the two cards from the draw pile, and continues their turn. If the cards don't match, the player flips the cards back over and ends their turn.

**How to win:** The game ends when the last pair is taken. The player with the most cards wins.

### Variations

- Allow at most one match per turn.
- Use a target sum larger than the largest card. Two cards match if their sum is the target.

## Bingo with Cards .....

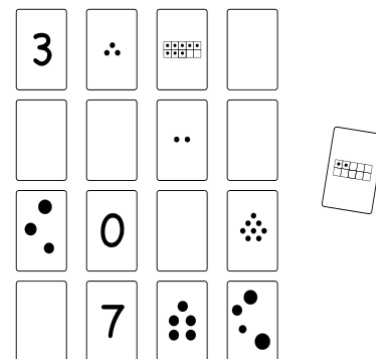


**The setup:** Play this with Number Cards, or if there are many players, with numbered regular playing cards.

**How to play:** Reserve two suits worth of cards for a draw pile, and divide the remaining cards among the players. Each player randomly selects 16 cards to place in a 4 by 4 grid face up in front of them.

Take a card from the draw pile and call it out. Each player may turn over one card from their grid that matches the number drawn. If a player has more than one card that matches, the player must choose which one to turn over.

**How to win:** The first player to get four cards flipped over in a row horizontally, vertically, or diagonally wins the game and calls out "Bingo!"



# DOMINO GAMES

**Prerequisite:** Counting up to 5; Dominoes Cards or commercial Dominoes Set

## Stealing Bones . . . . .

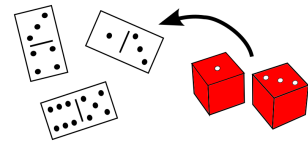


Game

**How to play:** Place all dominoes (bones) face up between the players. For a turn, a player rolls two dice. If the domino that matches these dice is available, the player claims it and moves it to their pile.

**How to win:** For two players, the first player with ten dominoes wins. For more than two players, the first player with six dominoes wins.

**Variation:** Claimed dominoes can still be stolen by other players.



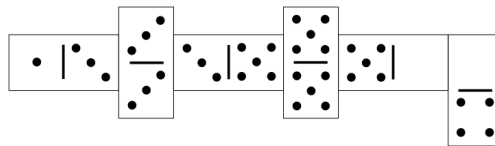
## Dominoes . . . . .



Game

There are many variations to this ancient game – discuss the rules before you start!

**The setup.** Mix up all tiles (bones) face down – this is called the boneyard. Take seven tiles each for two players, and five tiles each for three or four players. The player with the highest double starts by putting it on the table. If no one has a double, start the game over.



**How to play.** A turn consists of placing a tile that matches one of the available ends of the string of dominoes (place double dominoes crosswise to the previous domino). If the player has no matching tile, the player is “blocked,” and there are two possible rules to use: 1) the player’s turn is over, or 2) the player keeps drawing tiles until there is a match (one version restricts this to a single draw whether or not there is a match). If the boneyard is emptied before there is a move, play passes to the next player on the right.

**How to win:** The game ends when a player runs out of bones, or when all players are blocked. The winner is the player with the lowest total of dots on their tiles (which is 0 if they have used up all their tiles).

**Scoring options:** The winner’s score can be calculated in two ways. The traditional way is to use the sum of the dots on all the other players’ tiles. Rounds are played until a player reaches 50 or 100. For little ones, a better scoring system is to let the winner have the score which is the count of all the other players’ tiles.

**Variation:** When your child starts adding, change the rule for matching – two tiles match if their numbers add up to six (or nine if using a larger range of dominoes).

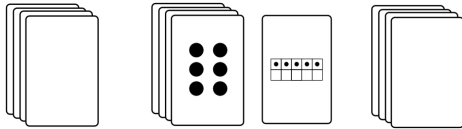
# NEARBY NUMBERS

**Prerequisite:** Counting up to 10

## Within One or Two .....



**The setup:** Create a stack of Number Cards with a range of quantities your child is comfortable with. Split the cards evenly between two players and place the cards face down.



**How to play:** Players take turns placing a single card on the stack in the middle between them. If the card is one more, the same, or one less than the previous card, the first player to say 'one more', 'the same' or 'one less' wins all the cards in the current stack.

**How to win:** The person wins who has the larger stack of cards when you stop playing.

### Variations

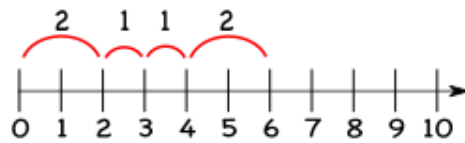
- Include the relationships of 'two more' and 'two less' as options.
- Use a target sum, say 10. Be the first to say the top two cards add up to the target.

## Nim with One and Two .....



**The setup:** Choose a target number, say 10. Let your child choose to go first or second.

**How to play:** Start at 0. Players take turns adding 1 or 2 to the current total. Count aloud the progress for each turn.



**How to win:** The player who lands on the target (e.g. 10) wins.

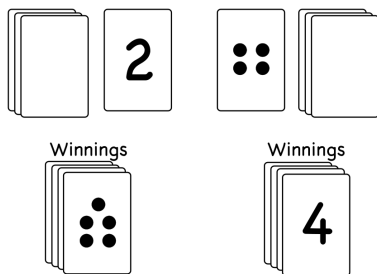
### Variations

- Once children learn to play this verbally, it is a great travel game.
- Use a pile of objects. Players add one or two to the pile until the target is reached.
- Use a number line. Advance a marker along the line one or two spaces for a move.
- Use subtraction. Players start at the target, say 10. On their turn, players choose whether to subtract 1 or 2. The first person to reach 0 wins.
- Use larger target numbers as your child's skills improve.
- Instead of winning, the player forced to hit or go beyond the target number loses.
- Allow a player to add (or subtract) 1, 2, or 3 for each turn.

# COMPARING

*Prerequisite: Counting up to 10*

## War – Single-Digit Comparing . . . . .



**The setup:** Remove the cards from two, four, or six Number Decks outside your child’s comfort range. Split the cards evenly in two piles face down.

**How to play:** Flip over the top cards and the player with the larger card keeps both cards. If the cards match, flip the next two cards and the winner gets all four cards.

**How to win:** The player with the most cards after one or more runs through all the cards is the winner.

**Variations:** To change things up, sometimes play that the smaller of the two cards wins.

## I'm Thinking of a Number . . . . .



This is an introductory version of a fun game we will revisit again soon.

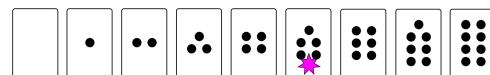
**How to play:** There are two people: the Puzzler thinks of a number and the Questioner discovers the number. The Puzzler declares, “I am thinking of a number from 0 to 8” (for example). The Questioner then asks questions of the form “How does your number compare to 3?” The Puzzler answers that the number is smaller, equal to, or larger than that number.

**Use cards:** With younger children, play this game with counting cards from 0 to 8 placed face up. Hide a star under one of the cards. After each guess, the Questioner turns over all the eliminated cards until the star is discovered.

**Example:** Here is a game with a target of 5:

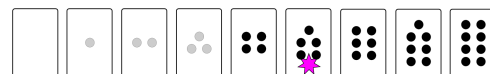
Puzzler: I’m thinking of a number from 0 to 8.

Question: How does your number compare to 3?



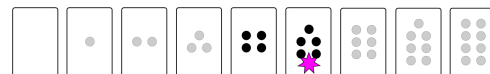
Puzzler: My number is larger than 3.

Question: How does your number compare to 6?



Puzzler: My number is smaller than 6.

Question: How does your number compare to 5?



Puzzler: Well done! My number is equal to 5.

# BIGGER AND SMALLER NUMBERS

*Prerequisite: Comfort with basic properties of objects; Shape Cards*

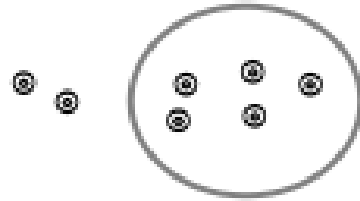
## Fitting In .....



Activity

Give your child a small number, say seven, of little objects and count them together. Pick a target number, say five.

Ask your child to take five objects out of the original group of seven objects, if it is possible. This helps your child learn the relative sizes of numbers and how much they are bigger or less than each other.



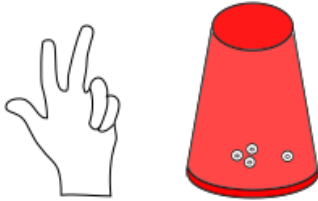
### Variations

*At first, the target number should be less than or equal to the total number. Later on, giving numbers that are too big provides practice with the relative size of quantities, as well as providing practice checking that requests are reasonable.*

## Invisible Adding and Subtracting .....



Activity



**First:** Have your child count a small number of objects and put them in a box.

**Next:** Ask them to put up the same number of fingers as there are objects in the box.

**Finally:** Show your child that you are adding (or removing) one or two objects to the box, and ask 'How many objects are in the box now?'

### Variations

*When this becomes too easy, you can add or remove more than two objects.*

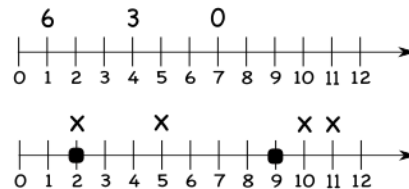
# BIGGER AND SMALLER NUMBERS

*Prerequisite: Comfort with basic properties of objects; Shape Cards*

## Number Line Battleship . . . . .



**The setup:** Each player has two number lines – one for their battleships and one for their guesses. These number lines go from 0 to 12 (or higher if the children can count higher). Out of sight, on their battleship number line, each player places tokens on two numbers that will be their battleships.



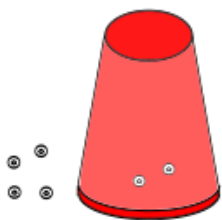
**How to play:** After the setup, the players take turns guessing numbers. When a player makes a guess, the other player says how close the guess was to the nearest target - the guesser then marks down that information on their second number line.

**How to win:** The first person to get all the targets wins.

### Variations

- Use a wider range of numbers.
- The response to a guess can be a range of distances rather than an exact amount - for example: "the nearest ship is 1 or 2 away."
- Have ships that are two or three spaces in length.

## What's Missing? . . . . .



Count a small number of little things. Hide a few while your child looks away. When your child looks back, ask how many are hidden.

**Example:** Suppose you have 6 raisins on a table. Have your child look away and cover 2 of them with a bowl. When your child looks back, count out the 4 visible raisins and ask how many raisins are under the bowl if there are 6 in total.

### Reasoning

One way for your child to figure this out is to "count on" from 4 to 6 - as your child counts 4, 5, and 6, start with 0 fingers raised and raise one finger at a time to arrive at 2 raised fingers. Similarly, your child can do almost the same thing by "counting down" from 6 to 4. Seeing that 4 plus 2 more is 6 is tied together with taking 2 away from 6 to get 4 is great practice for understanding fact families.

# SHAPE GAMES

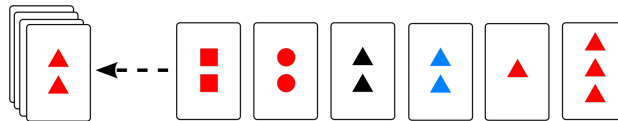
*Prerequisite: Comfort with basic properties of objects; Shape Cards*

## Feature Matching Game .....



Game

Shape Cards have three properties: shape (circle, triangle, square); count (one, two, three); color (red, blue, black).



**The setup:** Deal five cards to each player. Put the remaining cards face-down in a draw pile. Turn the draw pile's top card face-up to begin a new stack. Take turns placing a card on the stack – the new card must match two features of the top card. If you cannot play a card on your turn, draw a card from the draw pile and end your turn.

**How to win:** The first person to run out of cards is the winner. If the draw pile runs out, the player with the fewest cards in their hand wins.

### Variations

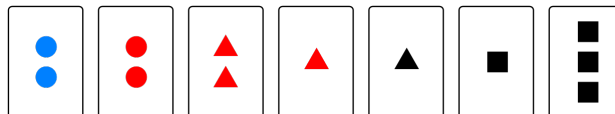
- Simplify the game by only requiring a match of one property with the top card.
- Allow a player to place more than one card on a turn as long as each card is a legal play on top of the previous card.

## Feature Matching Puzzle .....



Puzzle

Choose a Shape Card as your starting card. In our example, this is the card with two blue circles. Create a sequence of 4 to 8 cards that can legally be played - each card must share two features with the previous card.



Put the unused cards aside, separate the starting card, and shuffle the puzzle cards.

**The challenge:** Take the shuffled cards and place them in a series of legal moves on the starting card.

# SHAPE GAMES

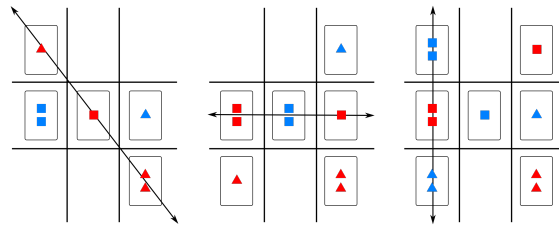
**Prerequisite:** *Comfort with basic properties of objects; Shape Cards*

## Triplet .....



**The setup:** Make a tic-tac-toe grid big enough to hold a Shape Card in each square. Place around the board 8 of the Shape Cards that have two of each property. For example, pick the 8 cards that are either a triangle or square, have one or two figures, and are red or blue.

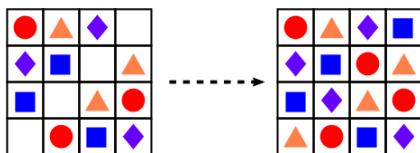
**How to play:** Players take turns picking an unused card for their opponent to place on the board. The cards on the board may be used by either play to get three in a row.



**How to win:** The player wins who places a card that completes 3 cards in a row with at least one property in common! If all the cards are played without a winner, it's a tie game.

**Variation:** Simplify the game and add an element of luck by placing the cards face down in a draw pile and having each player play the top card.

## Shape Sudoku .....



**The setup:** Use four tokens each of four different types. For example, use different colored gummy bears. We have used orange triangles, blue squares, red circles, and purple diamonds.

**How to create:** Create one of these puzzles by starting with the answer – this will be a pattern of tokens with one of each kind in each row and column, and one of each kind in each 2 by 2 corner box of the grid. Once you have the “answer,” pull off some of the tokens and place them in a pile to the side.

**The challenge:** Give the puzzle to your child to figure out how to put back the tokens that were pulled off.

**Puzzle creation strategies:** Here are some simple strategies for creating puzzles: remove one token from each row; remove all of one kind of token and one each of the other tokens; or remove one entire row and column.

# BEGINNING FINGER MATH

*Prerequisite: Can count up to 10; Number Cards*

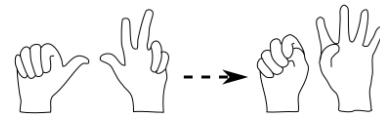
## Magic Hands – Add and Subtract . . . . .



Activity

This is a magic trick for practicing addition and subtraction. Do this slowly enough so that your child will get a chance to see what is going on.

**For addition:** Pick two numbers that add up to 5 or less. Count the number of raised fingers on your left hand, say 3. Count the number of raised fingers on your right hand, say 1. Bring the hands together and poof – the fingers on your right hand are transferred to the left – there are now 4 fingers raised. Magic!



Summarize by saying '3 fingers plus 1 more finger creates 4 fingers. Tada!'

**For subtraction:** Raise some fingers on your left hand, say 4 of them. Have your right hand reach over and grab some of the fingers, say 1 of them. Presto, there will be 3 fingers left raised on the left hand and 1 raised on the right hand. Summarize this saying that 4 take away 1 gives three, or say that you have broken 4 into two pieces which are 3 and 1. This wording emphasizes the concept of number bonds, which are pairs of numbers that add up to a given total.

### Important special cases

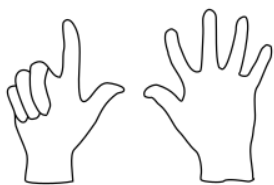
- *Addition: Let one or both of the hands have 0 fingers raised and show nothing changes when adding 0.*
- *Subtraction: Subtract all the fingers so none are left over, and sometimes subtract none of the fingers to show that nothing changes.*

## Flash Numbers . . . . .



Activity

Help your child to easily and quickly recognize numbers of objects. Practice in two ways.



**Use ten frames:** Use Number Cards that have ten frames. Pick a card at random and playfully see if your child can recognize the quantity. For variety, have your child quiz you sometimes.

**Use fingers:** Show some fingers on one or both hands and have your child recognize the total count. When two hands are used for numbers over five, one of the hands should have 5 fingers raised - that way it looks like a ten frame.

# FIGURES

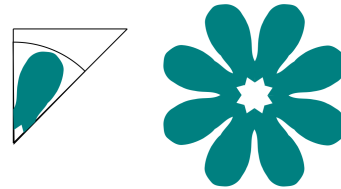
Prerequisite: Can count up to 10

## Cutting Symmetric Shapes . . . . .



Kirigami is the art of creating designs by folding a piece of paper and then cutting the paper while it is folded. There may be more than one fold, and the folds can happen in different directions.

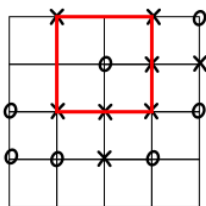
Folding the paper once and cutting it creates a design where one side is the mirror image, or reflection, of the other.



### Ideas to explore

- Experiment with cutting out faces, lamps, or geometric shapes.
- Use two intersecting folds to create designs with mirror images in two directions. This makes it easy to create designs such as flowers.
- Try various folds and cuts. Create snowflake designs by starting with the same two folds as in the last example. Then use two more folds that split the folded paper into thirds.
- Make a game of this by working backwards – draw a symmetrical shape on a piece of paper and challenge each other to cut a folded piece of paper to create that shape.

## Finding Squares . . . . .



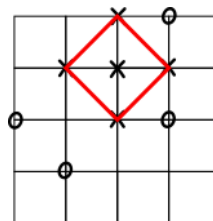
**The setup:** Create a 5 by 5 grid using five horizontal and vertical lines.

**How to play:** Players take turns putting their tokens on points where the lines cross in the grid.

**How to win:** The first player with four tokens on the corners of a square of any size wins.

### Variations

- Allow squares with diagonal sides.
- Use grids larger than 5 by 5.



# NUMBER STORIES

*Prerequisite: Counting to ten; Beginning skills with single-digit addition and subtraction*

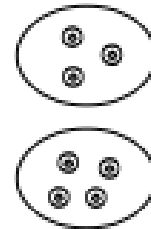
## Give Fun Names to Numbers . . . . .

Introduce the idea of variable names by using silly names for missing numbers in activities.

## Bag Stories – Add and Subtract . . . . .



**Stories about bags:** You and your child each pretend to have a bag with some number of things. One person creates a story such as: “Your bag has 3 raisins and mine has one more. How many do I have?” After your child gets comfortable, let your child come up with the question some of the time - this is often great fun for them, especially if you make the occasional “mistake.”



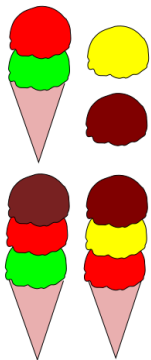
**Add complexity:** These stories can get more elaborate with experience. For example, the story could be “I have two fewer cookies than you do, and together we have six cookies. How many cookies do you have?” Another example, “You have twice as many candies as I do, and together we have nine pieces. How many pieces do you have?”

**Other story lines:** The use of bags and pieces of food can be replaced by the idea of a fish bowl with two (or more) kinds of fish, or with any other imagery that appeals to your child. For a fish bowl, you could create a story such as “There are seven fish in the bowl, and there is one more goldfish than there are tetras. How many goldfish are there?”

## Math Stories . . . . .



Stories are a fun way to add interest to calculations. Here are two examples of such stories:



**Chairs:** There are four chairs in a room. At first there are two people, but then three more come along. Can everyone sit down? This is more interesting than simply asking if  $2 + 3$  is larger than 4.

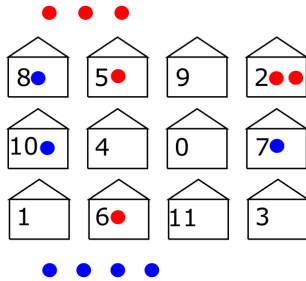
**Ice cream:** My two friends and I are having ice cream. I will give each of my friends one more scoop than I get. If there are ten scoops of ice cream, how many scoops of ice cream can I have?

**Variations:** Pick topics that interest your child, such as food or animals. As your child gets better at this, let some of the stories be somewhat vague so your child will need to analyze more and learn to ask clarifying questions.

# ORDER

**Prerequisite:** Can count forward and backward between 0 and 10; Number Cards, Number line

## Get Out of my House . . . . .



**The setup:** Use a deck of cards with numbers from 1 to 10. On a shared piece of paper, have boxes, or simple drawings of houses, numbered from 0 to 11. To provide practice figuring out order, do not put these boxes in order on the page. Each player has 7 tokens distinct from the other player's tokens – using different colors is one way to do this.

**How to play:** On a turn, a player picks a card and puts their token in any house that is one more or one less, as long as it does not have 3 or more of the other player's tokens in it already. If the house has one or two of the opponent's tokens, those are given back to the opponent and the player says "Get out of my house."

**How to win:** The first player to place all their tokens wins.

### Variations

- If not ready for numerals, use Number Cards and boxes with quantities of dots.
- Use a smaller or larger range of Number Cards and boxes.
- Allow moves to houses that are 2 numbers more or less.

## The In-between Game . . . . .



**The setup:** Use one set of cards from 0 to 10. Use either Number Cards or playing cards with Queen as 0 and Ace as 1. Each player also gets 20 tokens.

**How to play:** For the player whose turn it is, deal two cards face up and a third card face down between them. The player decides to bet 0 to 3 tokens that the third card is between the two cards. If the player is right, the player gets that many tokens from the other player. If the player is wrong, that many tokens go to the other player.



**How to win:** You can either play five rounds or until one player runs out of tokens. The player with the most tokens at the end wins.

# SUDOKU VARIATIONS

*Prerequisite: Counting forward and backward between 0 and 10*

## Number Sudoku . . . . .



**The setup:** These are similar to the Shape Sudokus, only now they use numbers (or quantities of dots). To avoid erasing, use numbered (or dotted) slips of paper to solve the puzzles.

1	2		4
	3	1	
2		4	
3		2	1

	3		
	4		2
2		4	
		1	

For a 4 by 4 puzzle, each row and column has the numbers from 1 to 4 once. Also, each marked subregion has the numbers from 1 to 4 once. That's it! Create these puzzles for your child by starting with a completed puzzle and removing some of the pieces of paper.

**Variations:** For variety, use irregularly-sized subregions – these are called Jigsaw Sudoku puzzles. You can also create puzzles of larger size (two 5 by 5 puzzles are shown).

1	3			
2				
			1	
		3	4	

3				
			4	
		1		
	2			
				5

## Sudoku – Comparing . . . . .



>	<	>	<
<	>	<	>
>	<	>	<
<	>	<	>



2	>	1	3	<	4
4	>	3	2	>	1
1	<	2	4	>	3
3	<	4	1	<	2

These puzzles start with the same rules as regular Sudoku – each number appears exactly once in each row, column, and subregion. Additionally, if there is a less than or greater than symbol between two cells, then the numbers in the cells must obey that relationship.

**Hungry alligator:** Tell a child who has never seen a comparison symbol,  $>$ , that the bigger number is on the side with the wider part of the symbol. Some people say that the symbol is a hungry alligator and it always wants its mouth to point in the direction of the larger number.

**Solving strategy:** First look for where the smallest and largest numbers are.

As your child gets better, make the puzzles more challenging by leaving out more of the inequality symbols.

**Puzzle creation:** Make these puzzles by using a finished Sudoku puzzle. Put in greater than and less than signs on a blank grid of the same geometry. If your child gets stuck, put in a few numbers to get them started.

<	>	<	>
<	>	<	>
<	>	<	>
<	>	<	>



4	<	5	>	2	1	<	6	>	3
6	>	1	<	3	5	>	4	>	2
1	<	6	>	4	2	<	3	<	5
3	>	2	<	5	6	>	1	<	4
5	>	3	>	1	4	>	2	<	6
2	<	4	<	6	3	<	5	>	1

# BEING LOGICAL

**Prerequisite:** Counting to 10; Early logical and problem solving skills

## Make me a Liar .....



Activity

Someone makes a statement and the other players attempt to show that the statement maker is lying. The goal is to come up with a counterexample that shows that the statement is not always true.

**Always true:** One type of statement is saying something is always true. Examples are:

- All trucks have four wheels
- All rectangles are squares
- All birds can fly

**If -> then:** Another type of statement is of the form “if then”. Examples are:

- If today is Monday, then it is a school day
- If I don't eat for three hours, then I am hungry
- If a person is taller than someone, then they are older

## Codebreaker .....



Game

3	2	1
1	3	1
4	5	4
2	3	2
1	3	2
3	2	1

**The setup:** The CodeMaster creates a code, and the other player is the CodeBreaker. Suppose the code has three positions each of which can be from 1 to 5. An example of such a code would be 321.

**Breaking the code:** The CodeBreaker guesses a code and the CodeMaster says how close the guess is. For example, if the CodeBreaker guesses 131, then the CodeMaster would say that one spot was exactly right and one other spot had the correct number but in the wrong place. Play continues until the CodeBreaker figures out the code.

**How to win:** The number of guesses is the score for the CodeBreaker. The lowest score wins.

### Variations

- Add challenge by having a maximum number of questions that are allowed.
- Allow or disallow repeated numbers in the code.
- Use shorter or longer lengths for the code.
- Use a narrower or wider range of numbers for each place of the code.

# FINGER MATH

*Prerequisite: Counting forward and backward from 0 to 10*

## Finger Addition to 10 . . . . .



We'll use  $4 + 2$  to demonstrate two methods for this.

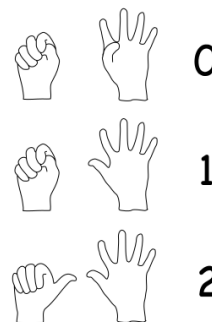


**Method 1:** Use this method if both numbers are 5 or less. Put up 4 fingers on one hand and 2 fingers on the other. Bring the two hands together - Tada! Your child sees that 4 and 2 brought together becomes 6.

**Method 2:** Use "counting on" from one of the numbers to reach any sum of up to 10.

**Example:** To add 4 and 2, put up 4 fingers on one hand, and count aloud from 0 to 2. For each spoken number after 0, put up one more finger. When 2 is reached, there should be 6 fingers raised.

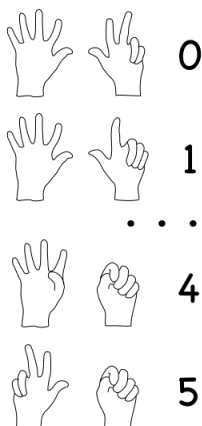
This method allows addition of numbers bigger than 5. Your child will come to realize that it's easier to start with the larger number of fingers raised and count on using the smaller number.



## Finger Subtraction to 10 . . . . .



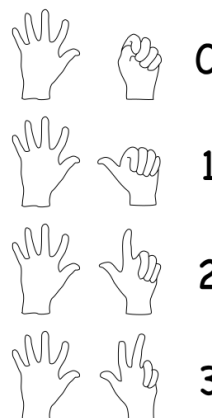
There are two mental models for subtraction - "take away" or "difference." Your child should be comfortable with both models. Here is  $8 - 5$  calculated both ways:



**Take away (on the left):** Think of  $8 - 5$  as what is left after you start with 8 things and take 5 of them away. Start with 8 fingers raised. Next, count aloud from 0 to 5, and for each number after 0 put one finger down. When 5 is reached in the count, there will be three fingers left up.

**Difference (on the right):** This model views  $8 - 5$  as finding the difference or distance between the two numbers. Start with 5 fingers raised. Then count the new fingers being raised, and when there are 8 fingers raised the difference of 3 will have been counted.

This method uses the "counting on" addition method to find the number to add to 5 to get 8.



Mix in problems where a number is subtracted from itself, and where 0 is being subtracted.

# NUMBER SIZES

**Prerequisite:** Can count to 10 or so and have a sense of their quantities.

## I'm Thinking of a Number . . . . .



**The setup:** There are two people - the Puzzler, who thinks of a number, and the Questioner, who discovers the number.

**How to play:** To start, the Puzzler says, "Bowser is a number from 0 to 12." The Questioner then asks questions of the form "How does Bowser compare to 4?" The Puzzler then says Bowser is smaller, equal to, or larger than 4. **Example:** The Puzzler thinks of 11. The discussion could go like this:

- Puzzler: Stripes is a number between 0 and 15.
- Questioner: How does Stripes compare to 8?
- Puzzler: Stripes is bigger than 8.
- Questioner: How does Stripes compare to 12?
- Puzzler: Stripes is less than 12.
- Questioner: How does Stripes compare to 10?
- Puzzler: Stripes is bigger than 10.
- Questioner: Is your number 11?
- Puzzler: Yes, congratulations!

### Variations

*Make this into a game by counting the questions. After alternating turns, the player asking the smaller total number of questions wins.*  
*As your child's mathematics develops, use other kinds of questions, such as "Is Bowser even?" or "Is Bowser a prime number?"*

## Estimation Game . . . . .



Develop a sense of quantities by seeing who can make the best estimate for the size of a group, such as a group of people standing in line.

Force yourselves to make a quick estimate without doing any counting. After everyone gives an estimate, count the objects and reward the person who is the closest.



# NUMBERS IN ORDER

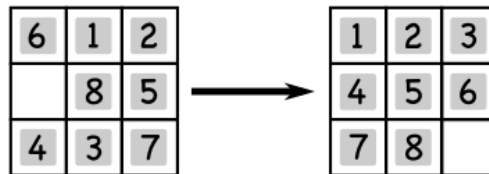
**Prerequisite:** Can count to 15

## 15 Sliding Puzzle .....



**The setup:** Start with a 4 x 4 empty grid of squares formed by 5 horizontal and vertical lines. Use a set of 15 pieces of paper the size of the grid squares, and number the pieces of paper from 1 to 15. The puzzle starts by having someone place the pieces of paper on the grid.

**The challenge:** The object of the puzzle is to get the pieces of paper in order with only the lower right hand corner of the grid empty. To achieve this, a piece of paper can be moved if it is adjacent to the empty square - in which case it can be slid into that space. Depending on how the person sets up the puzzle, the puzzle may or may not be solvable.



**How to create:** To create these puzzles, you have two options. The first is to place the squares randomly, in which case you have a 50 / 50 chance of the position being solvable. Alternatively, you can start by placing the pieces of paper in the final position and then making a series of legal moves to move the paper around. When you are all done, you are guaranteed that the puzzle is solvable.

### Different sizes

*If a 4 x 4 grid is too hard for a beginner, start with something smaller. The grid could be as small as 2 x 2 or as big as the child wants. The number of numbered pieces of paper will always be one less than the size of the grid. For example, on a 2 x 3 grid use the cards from 1 to 5.*